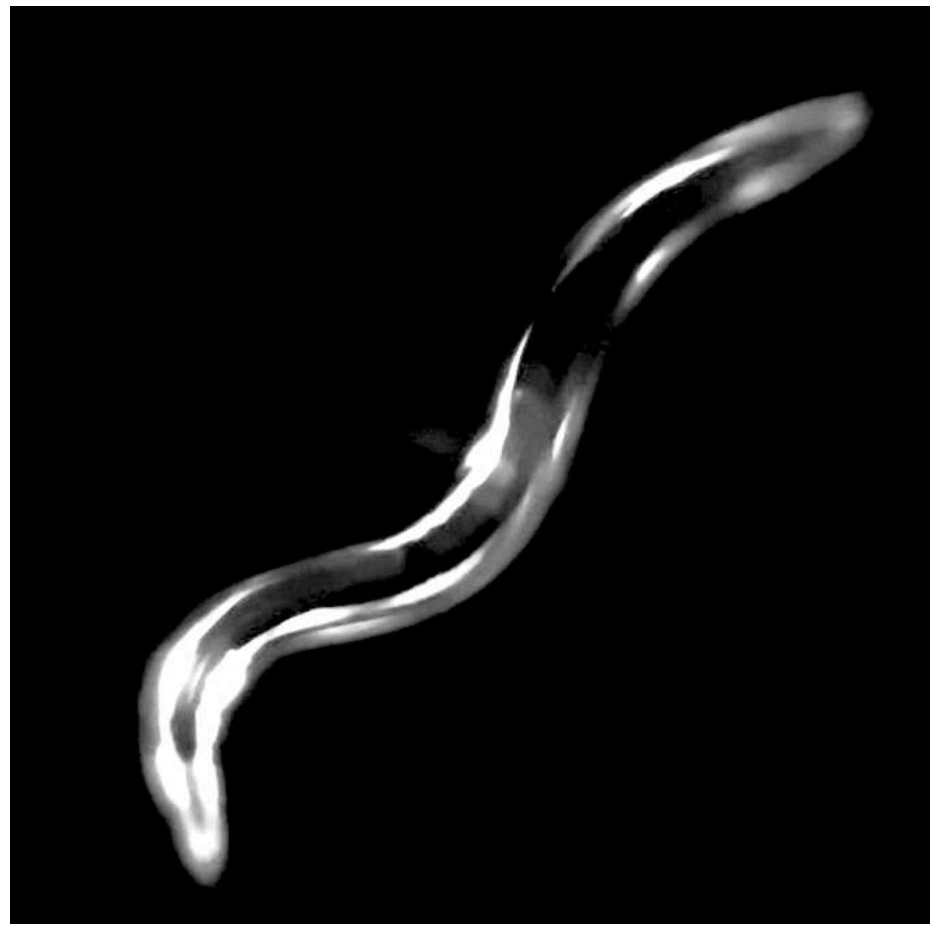
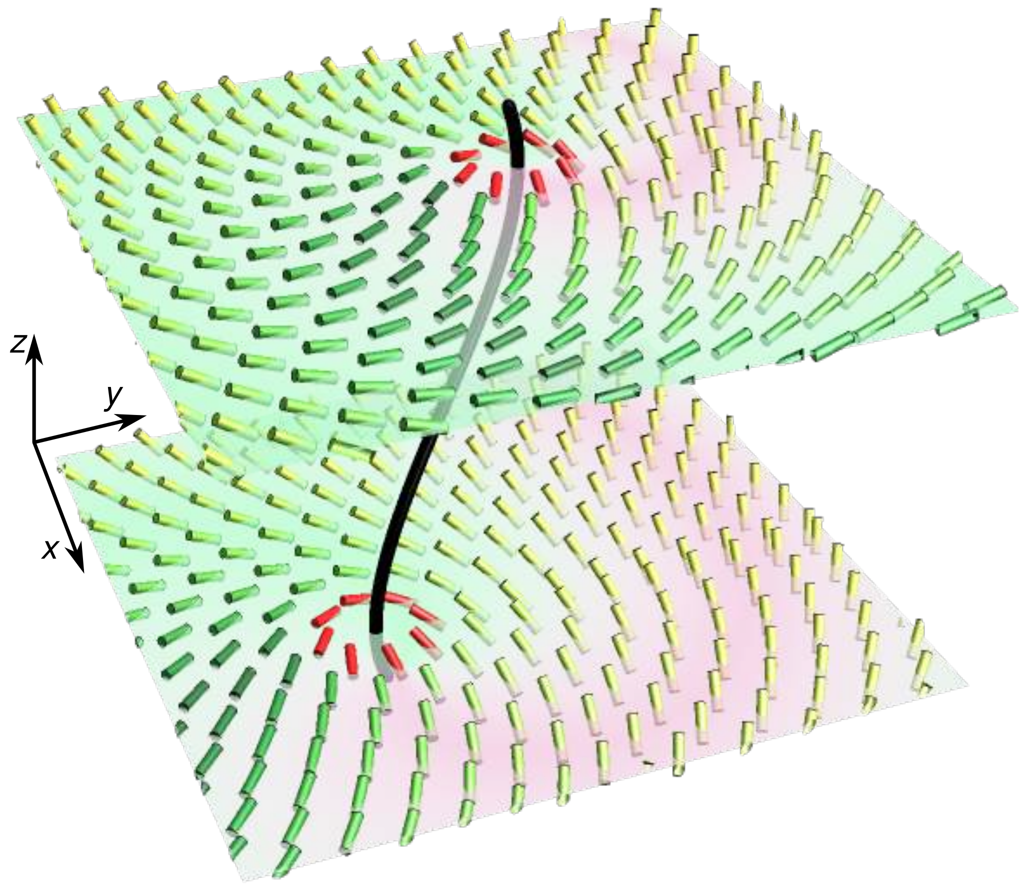
MIT MATHEMATICS MathWorks NSF Alfred P. Sloan FOUNDATION Schmidt Sciences

Quantum analogues in living and soft matter

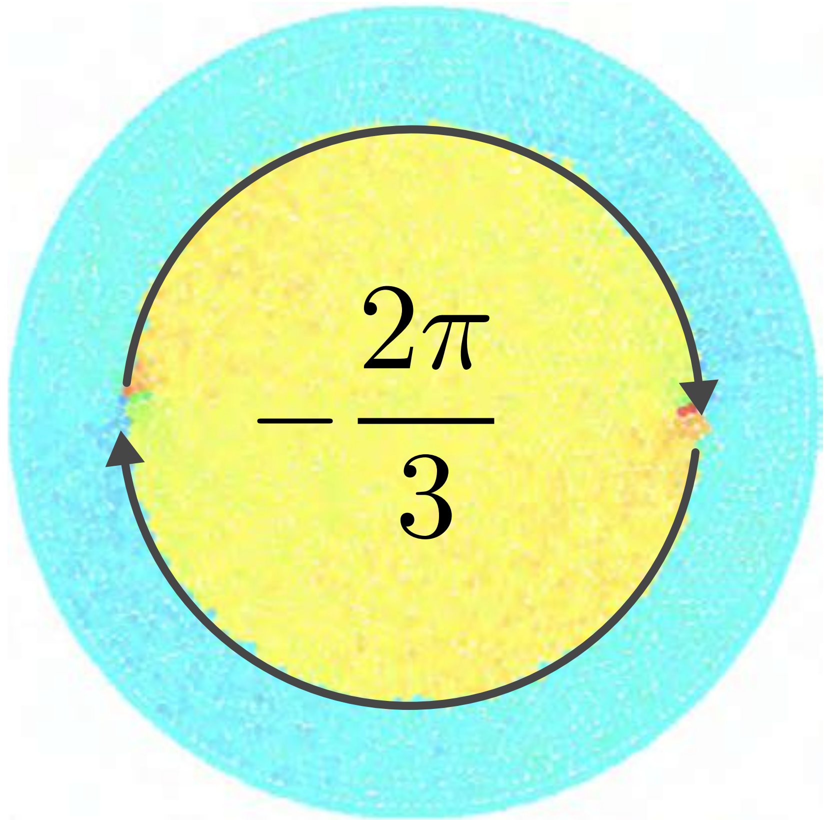


Flavell lab (MIT BCS)

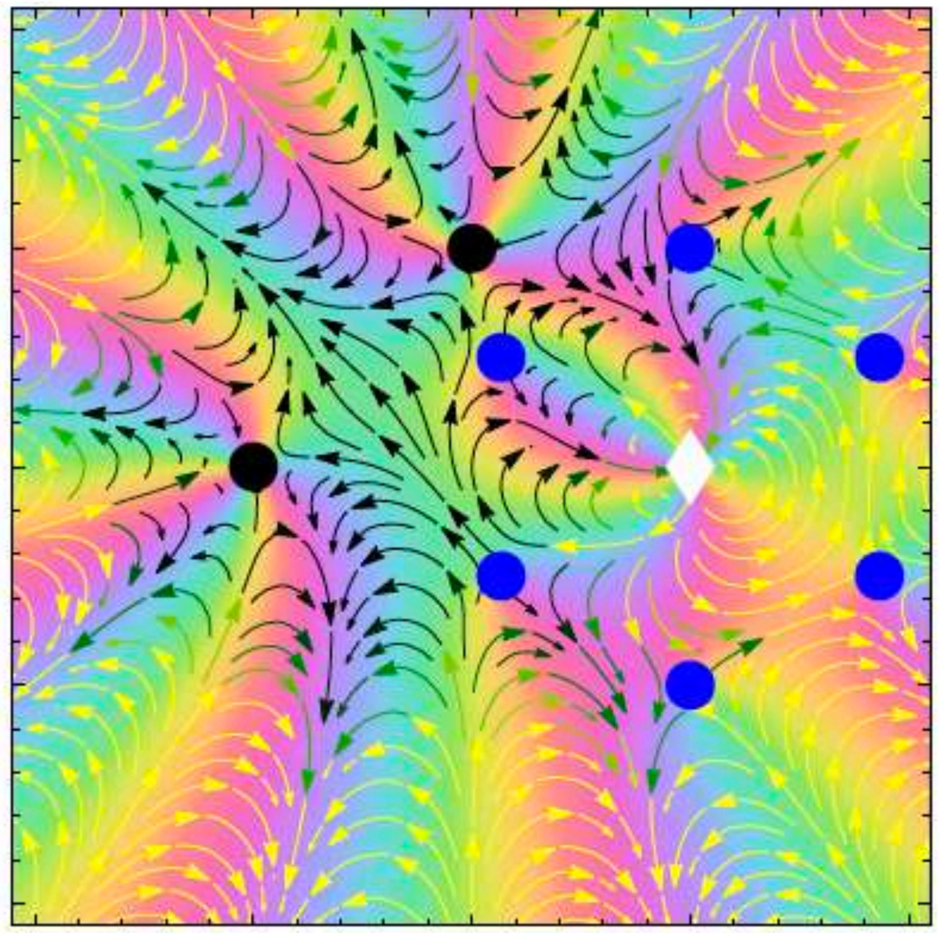
PRL 2023



Science Advances 2022



PRX 2022



$\frac{1}{\sqrt{2}}|11\rangle \otimes (|0\rangle + |1\rangle)$

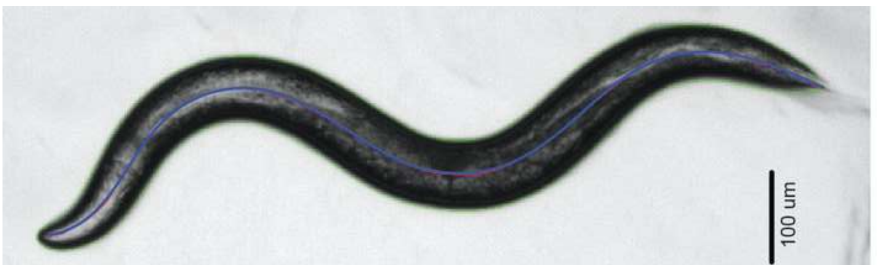
PRR 2024

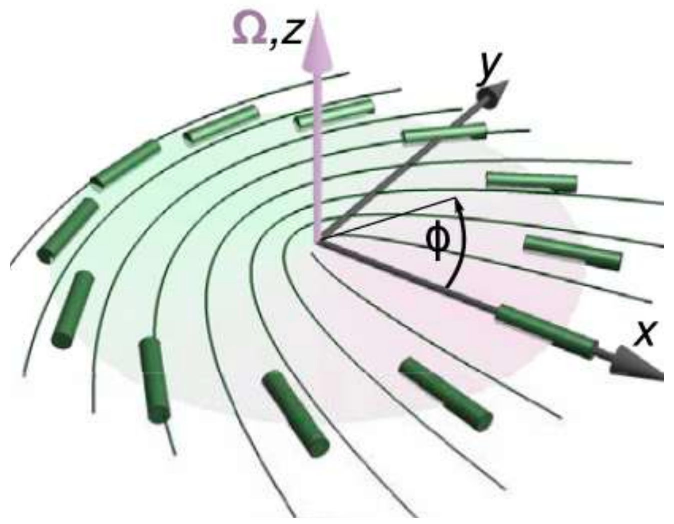

FIBA
EUROBASKET
DEUTSCHLAND
CHAMPIONS

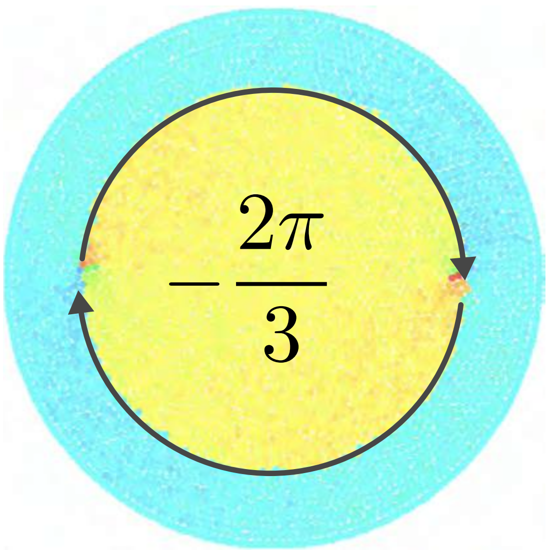
Outline

1. Dynamics of quasi-1D living systems
   - spectral representation & analysis
   - effective Schrödinger-like short-time dynamics

2. Topological defects & computation in liquid crystals
   - nematic bits
   - fractional defects and 'anyonic' braiding

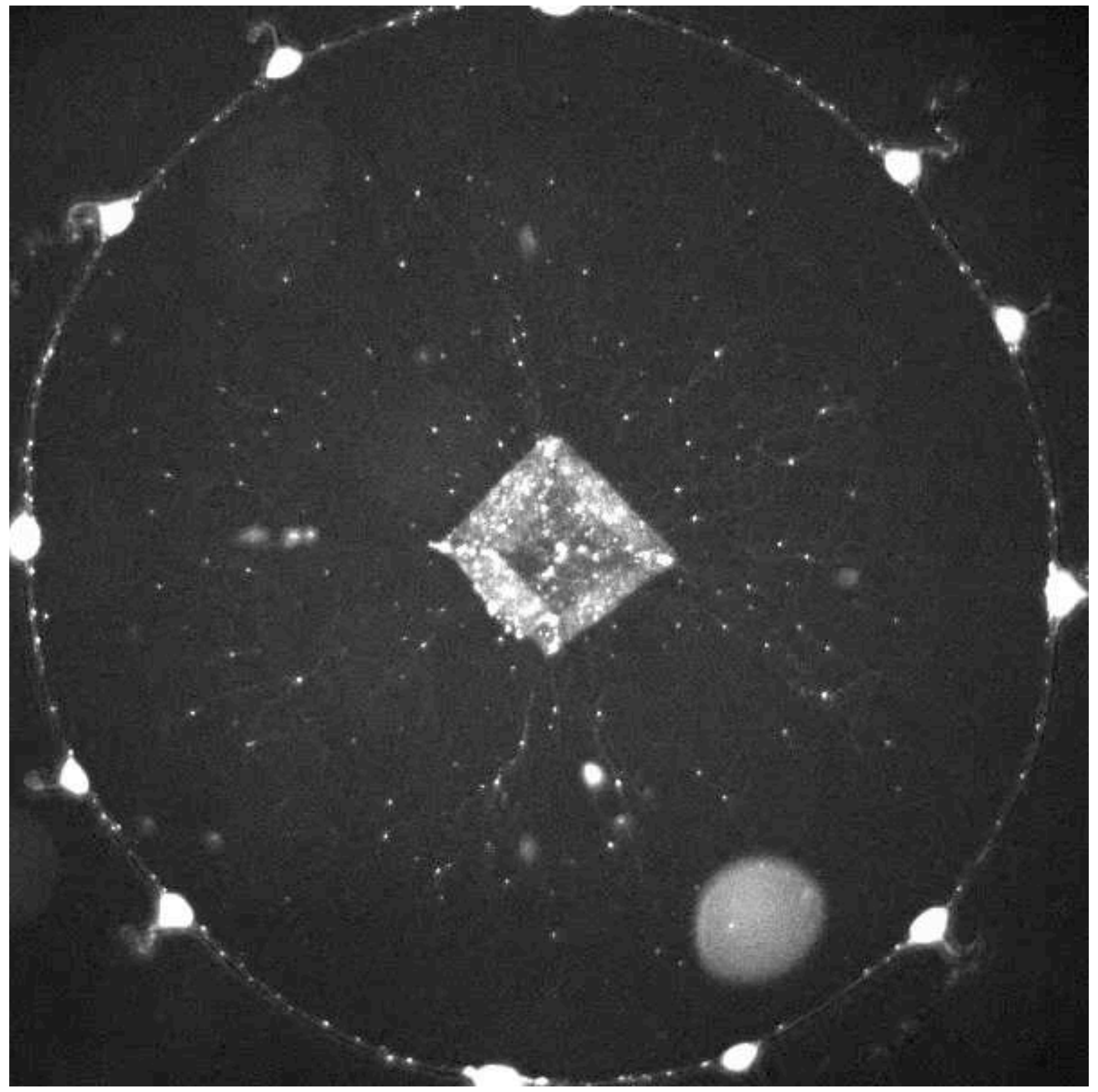






How do living and soft matter systems compute ?

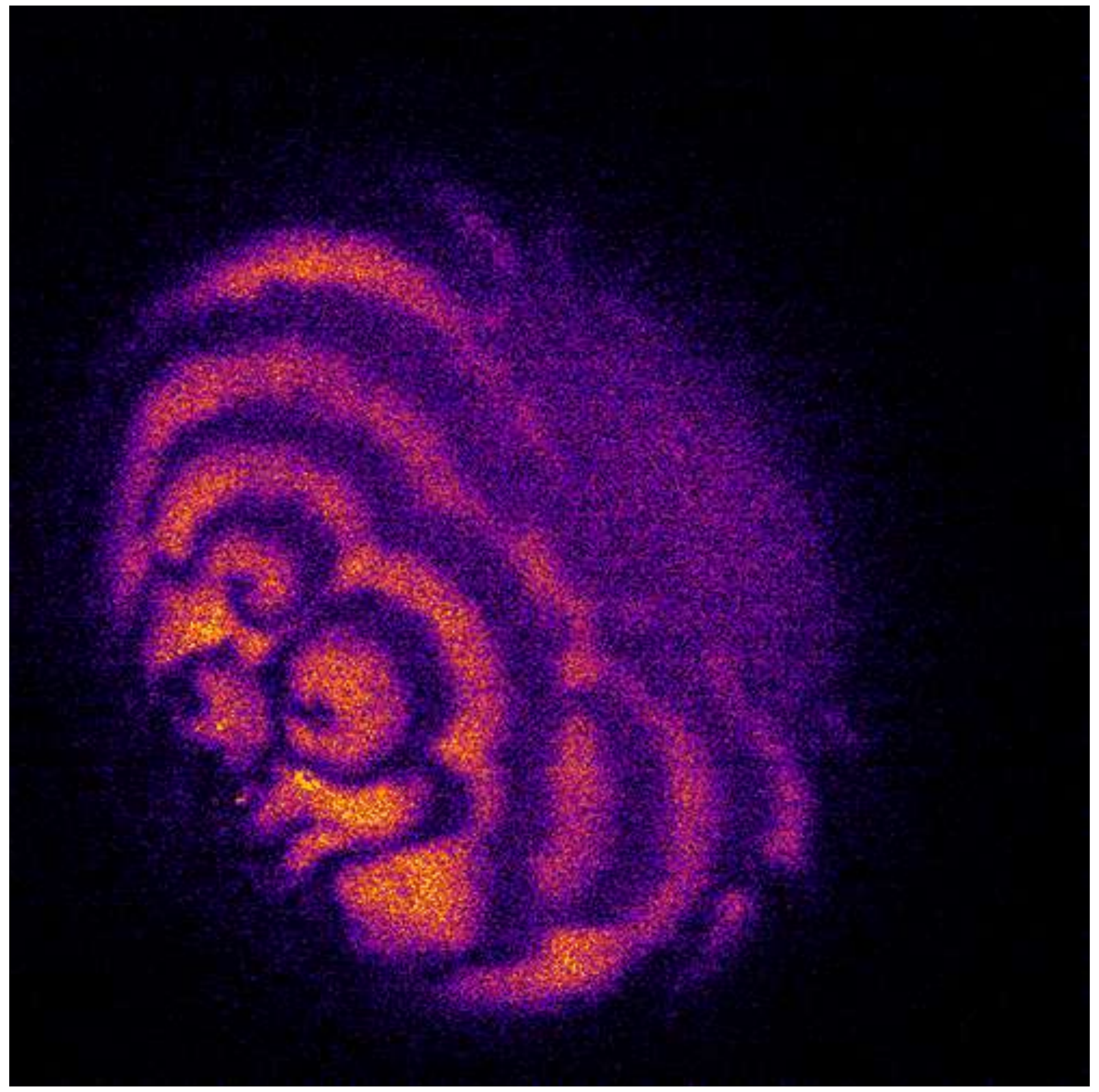
Jellyfish

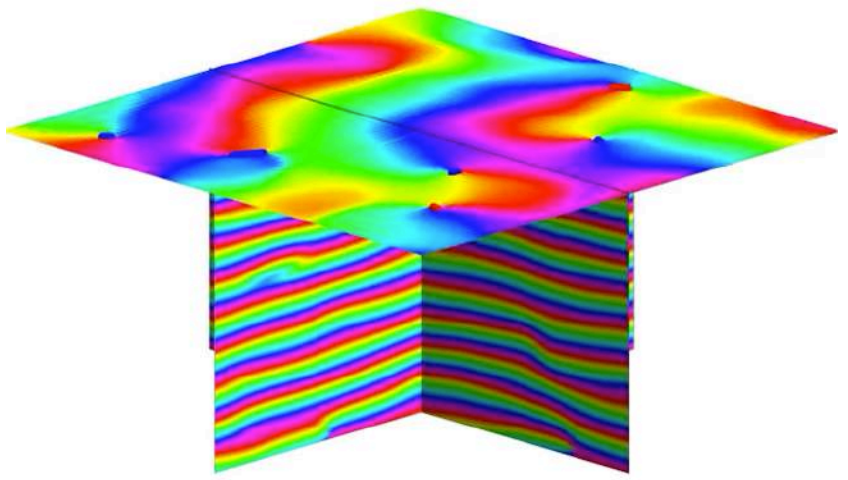


Weissbourd lab (MIT Biology)

In progress

Starfish oocyte



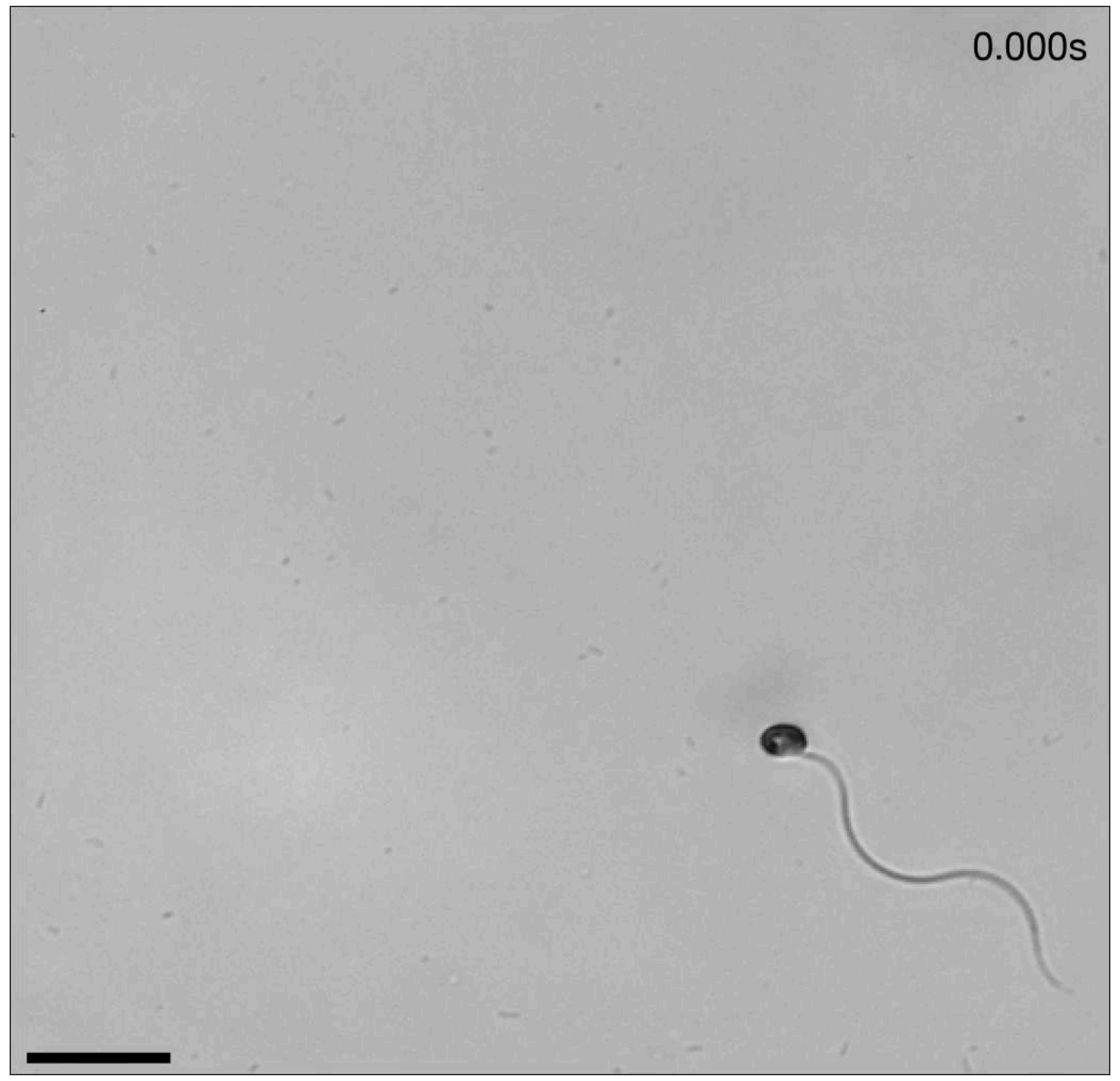
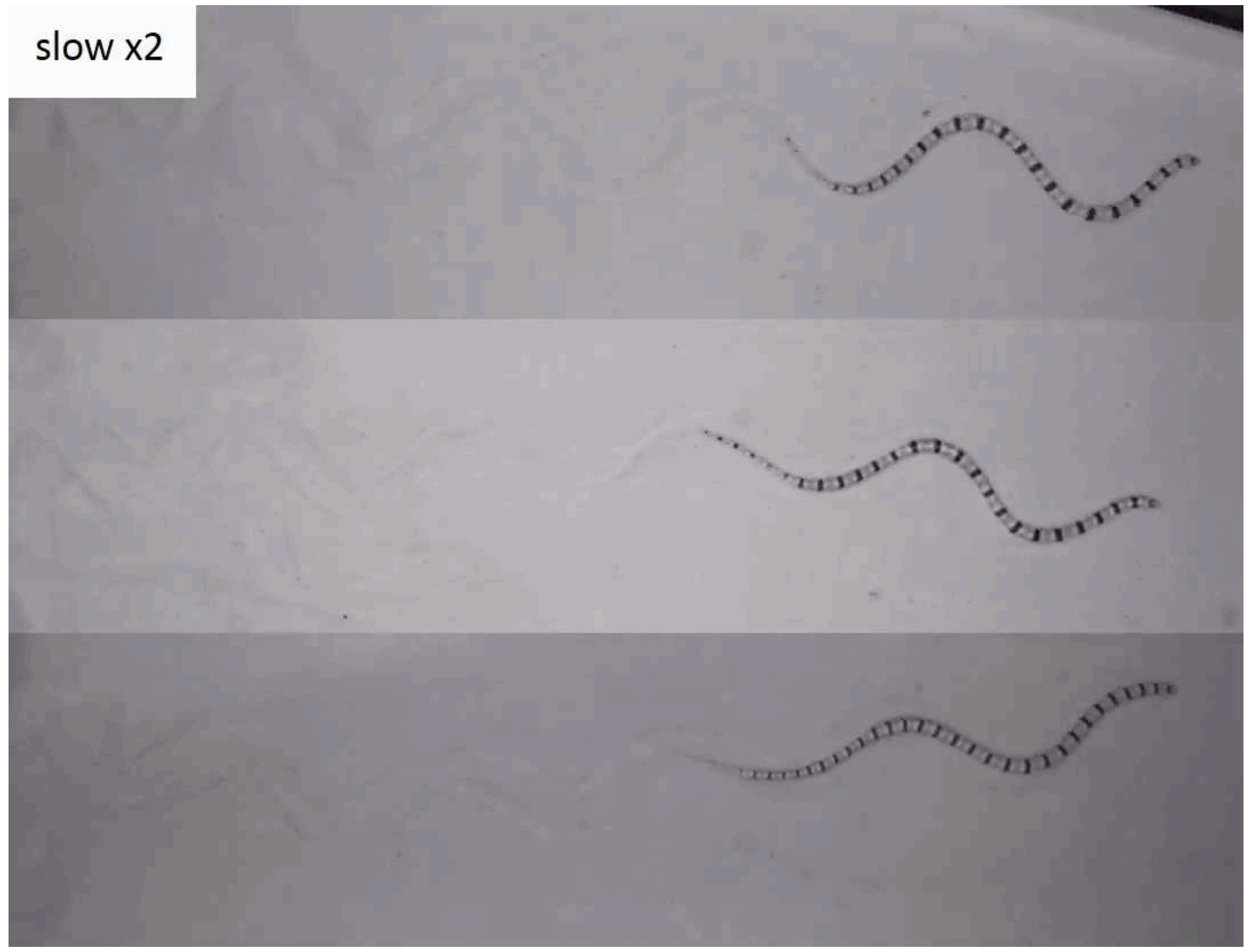


Fakhri lab (MIT Physics)

Nature Physics 2020
PNAS 2021

Part 1:

Spectral dynamics of quasi-1D living matter in 2D

Pterosperma alga	C elegans worm	Desert snake
		
Wang Lab (Exeter Uni)	Flavell Lab (MIT BCS)	Goldman Lab (Georgia Tech)
μm	mm	m

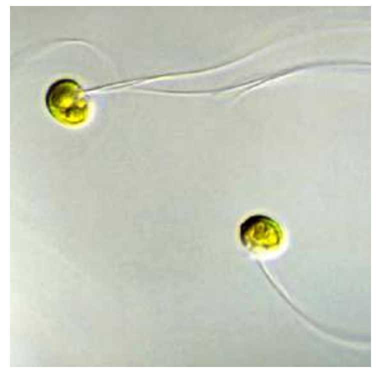
Would like to …

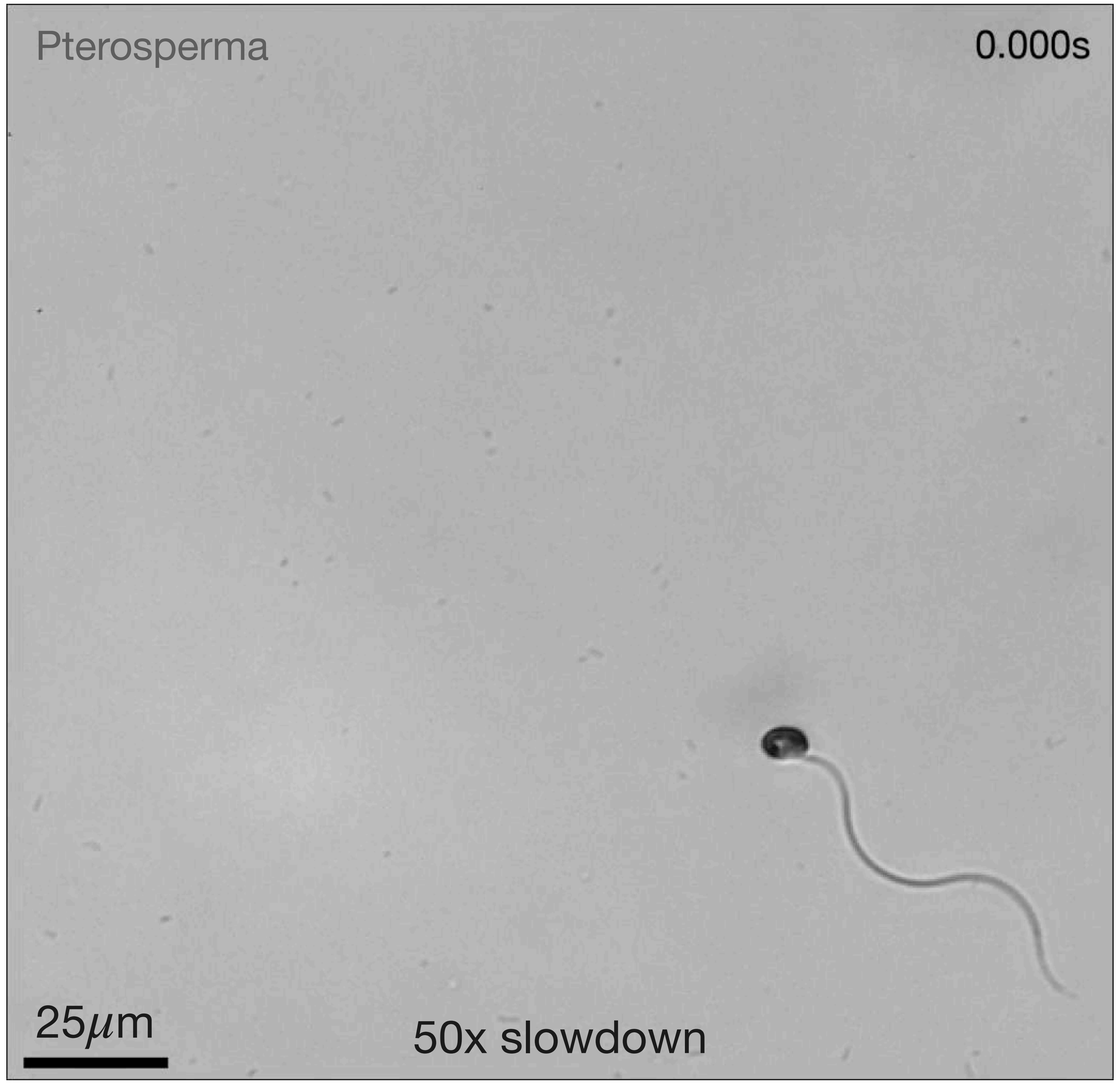
- Identify suitable collective (‘slow’) variables (low-dimensional representations)
- Identify and characterize different biological / biophysical states
- Infer dynamical models directly from video data
- Link biological signaling to physical dynamics

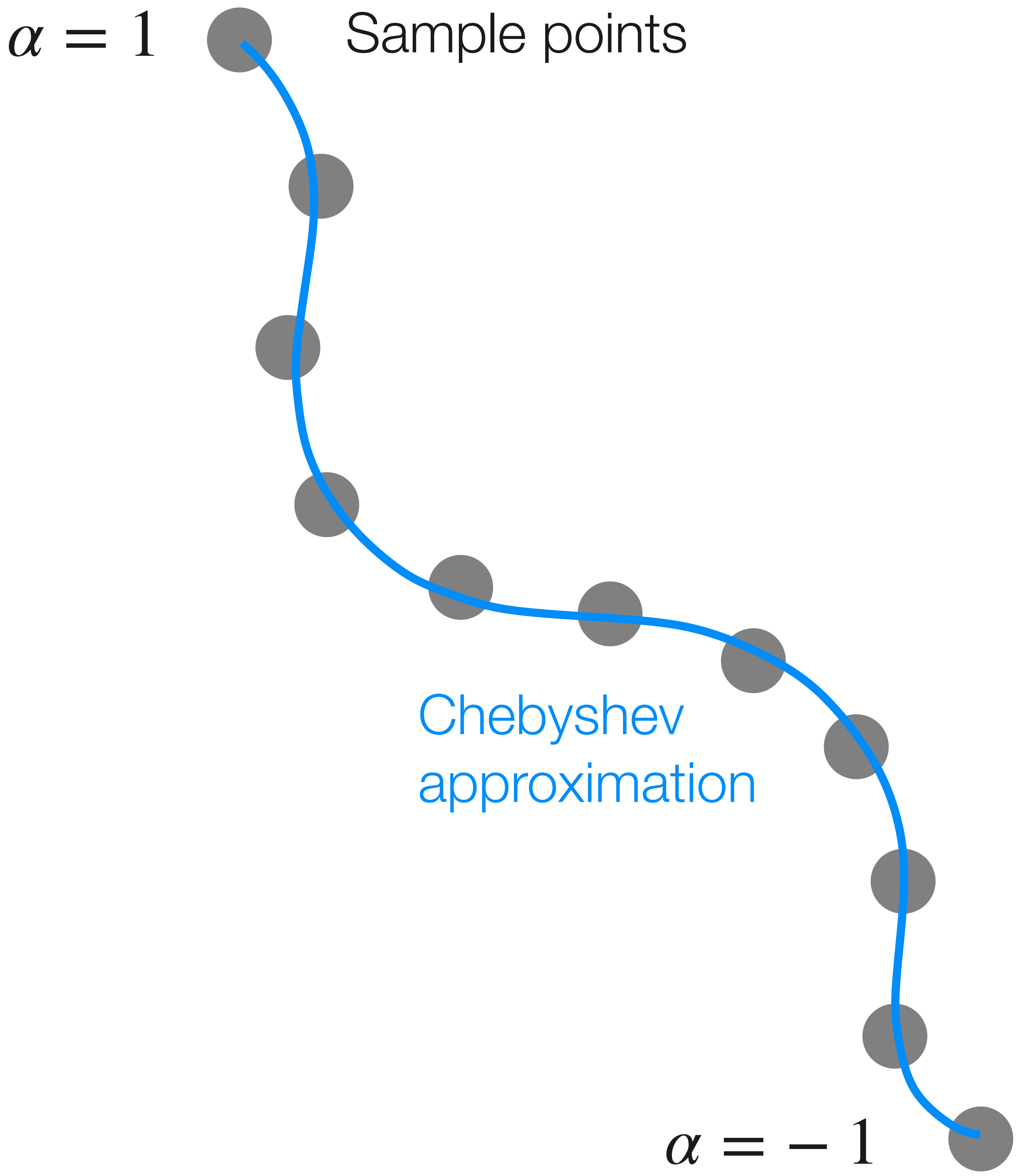
Spectral representation of biological dynamics & behaviors


Kirsty Wan

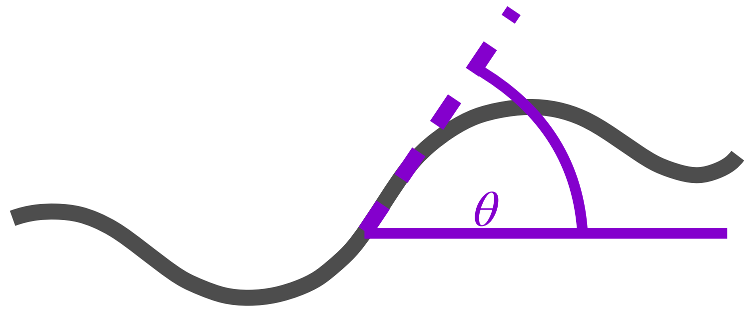

Alex Boggon

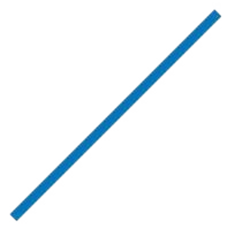
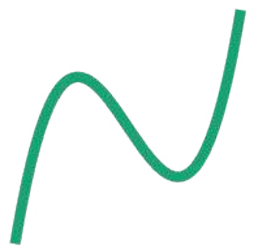




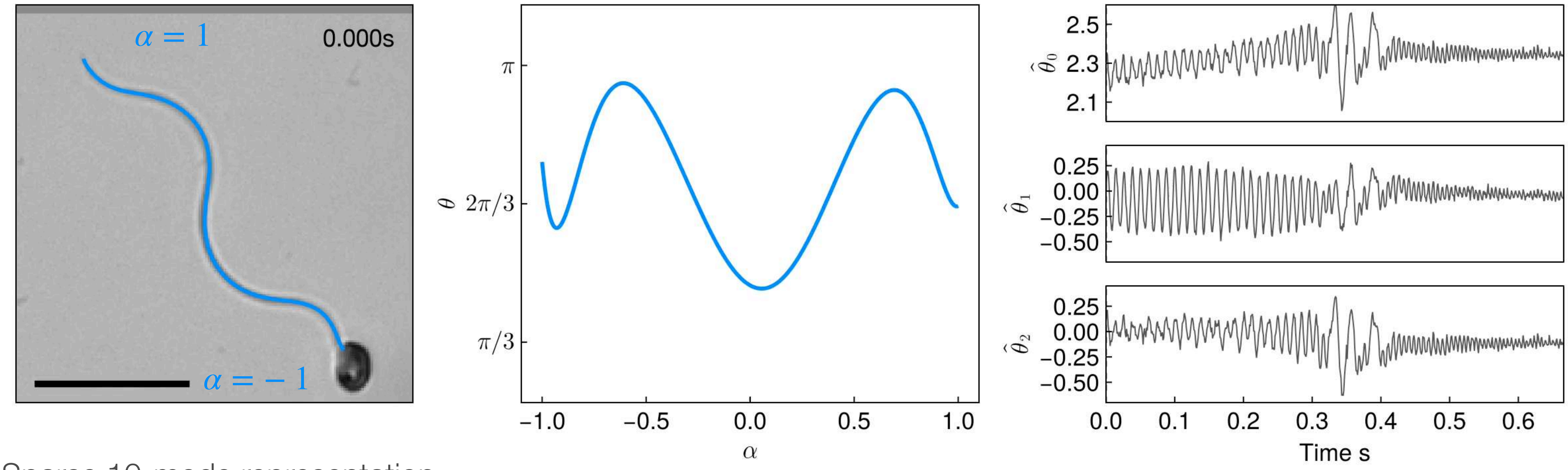


Spectral representation of biological dynamics & behaviors



$\theta(t,\alpha) \approx \sum_{n=1}^{N} \hat{\theta}_n(t) T_n(\alpha) = \hat{\theta}_1(t)$  $+\hat{\theta}_2(t)$  $+\hat{\theta}_3(t)$  $+\hat{\theta}_4(t)$  $+\ldots$

Orthogonal polynomial basis



Sparse 10-mode representation

Arts & Broek, Nat Comp Sci 2022

Wavelet analysis in time

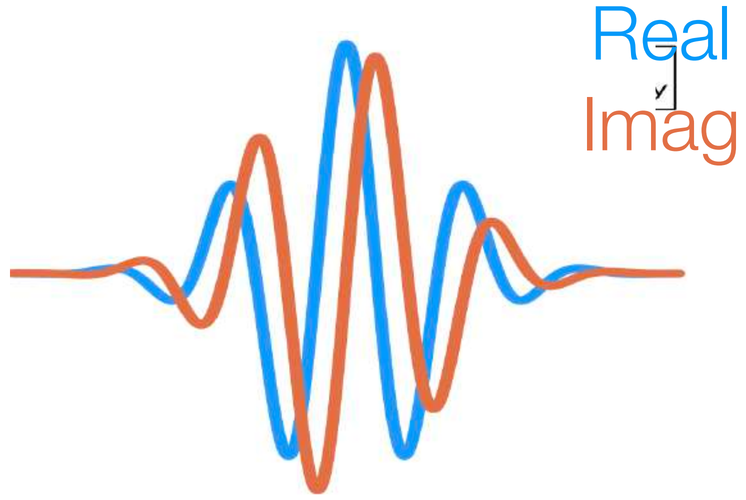
Convolve signal with shifts and scales of wavelet

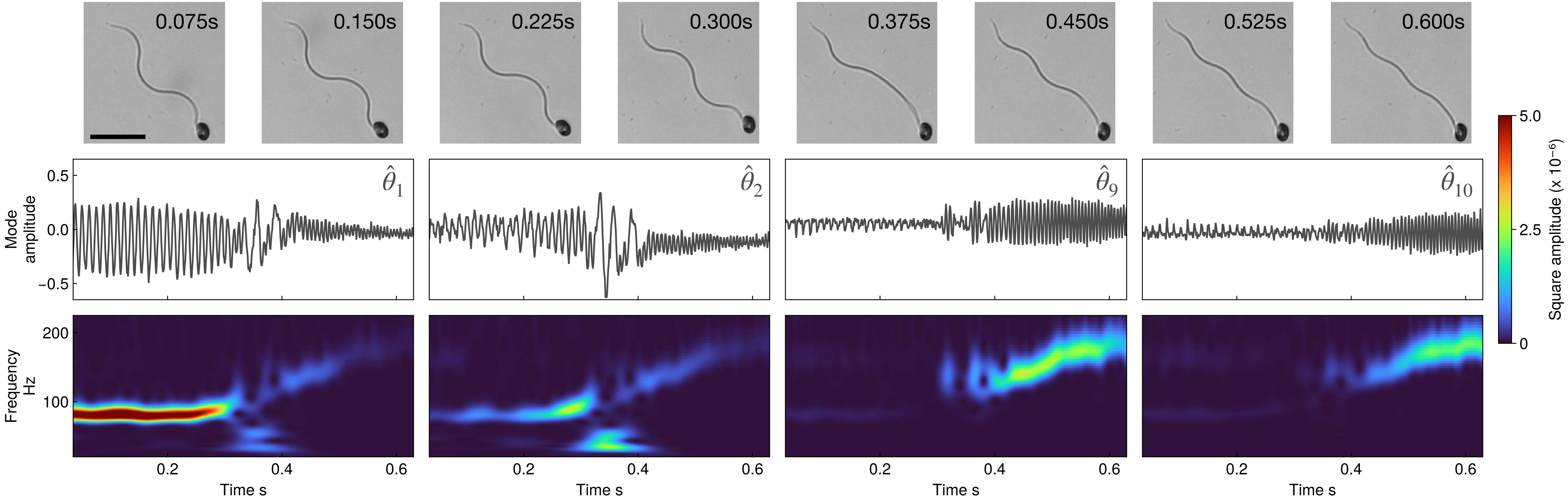
$$W_\psi[f](a,b) = \frac{1}{|a|}\int \mathrm{d}t\, f(t)\,\bar{\psi}\left(\frac{t-b}{a}\right)$$

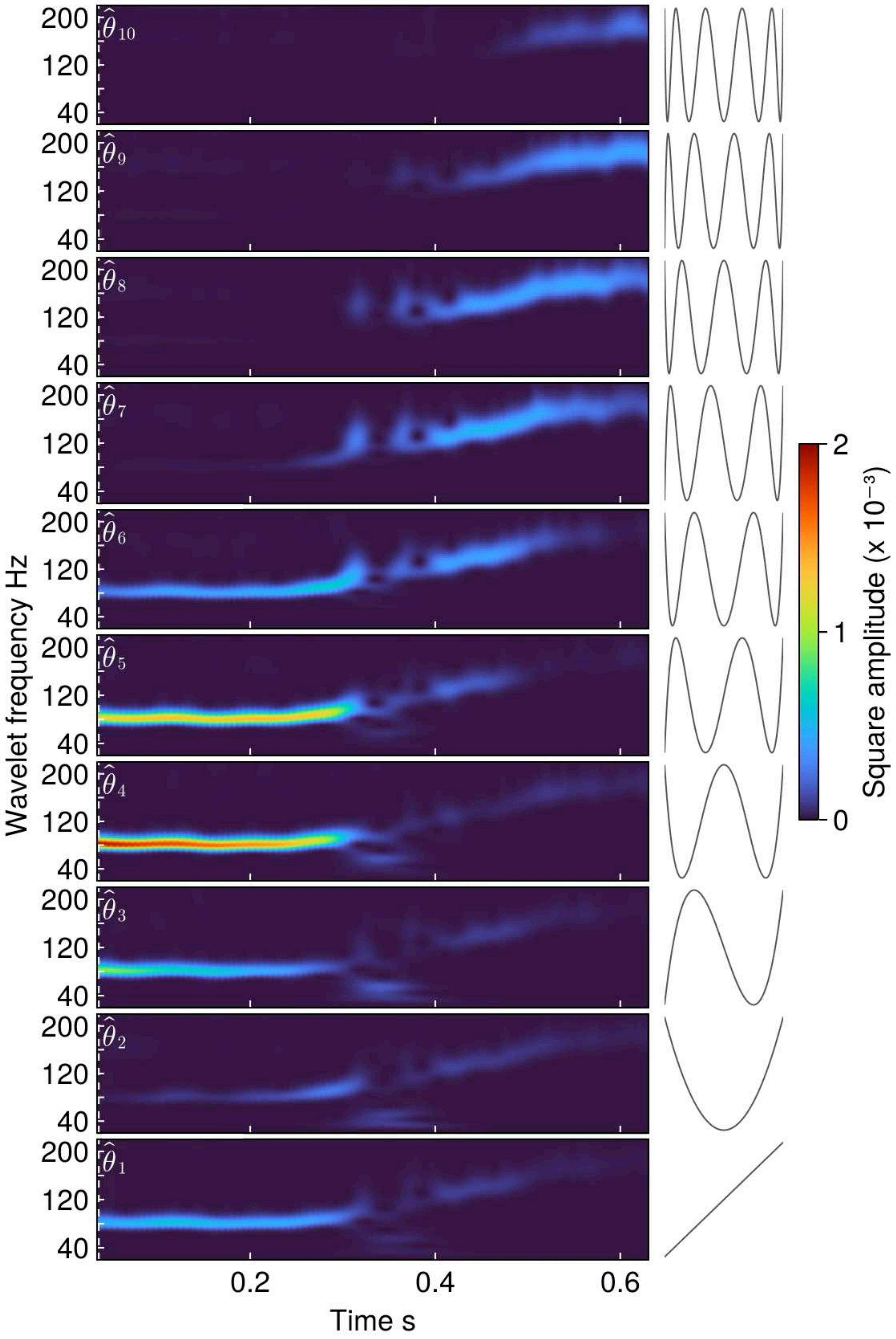
temporal location

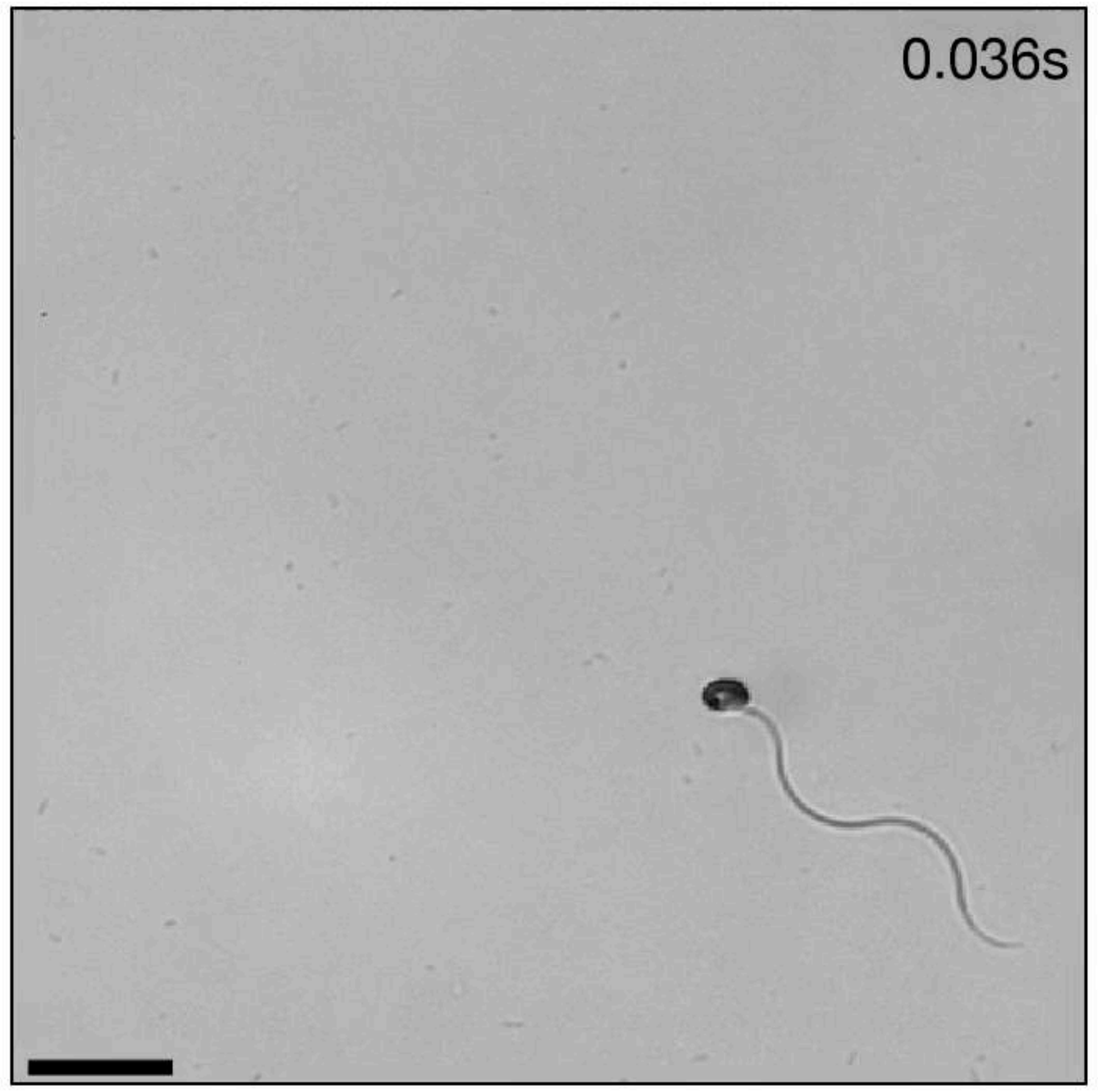
wavelet frequency

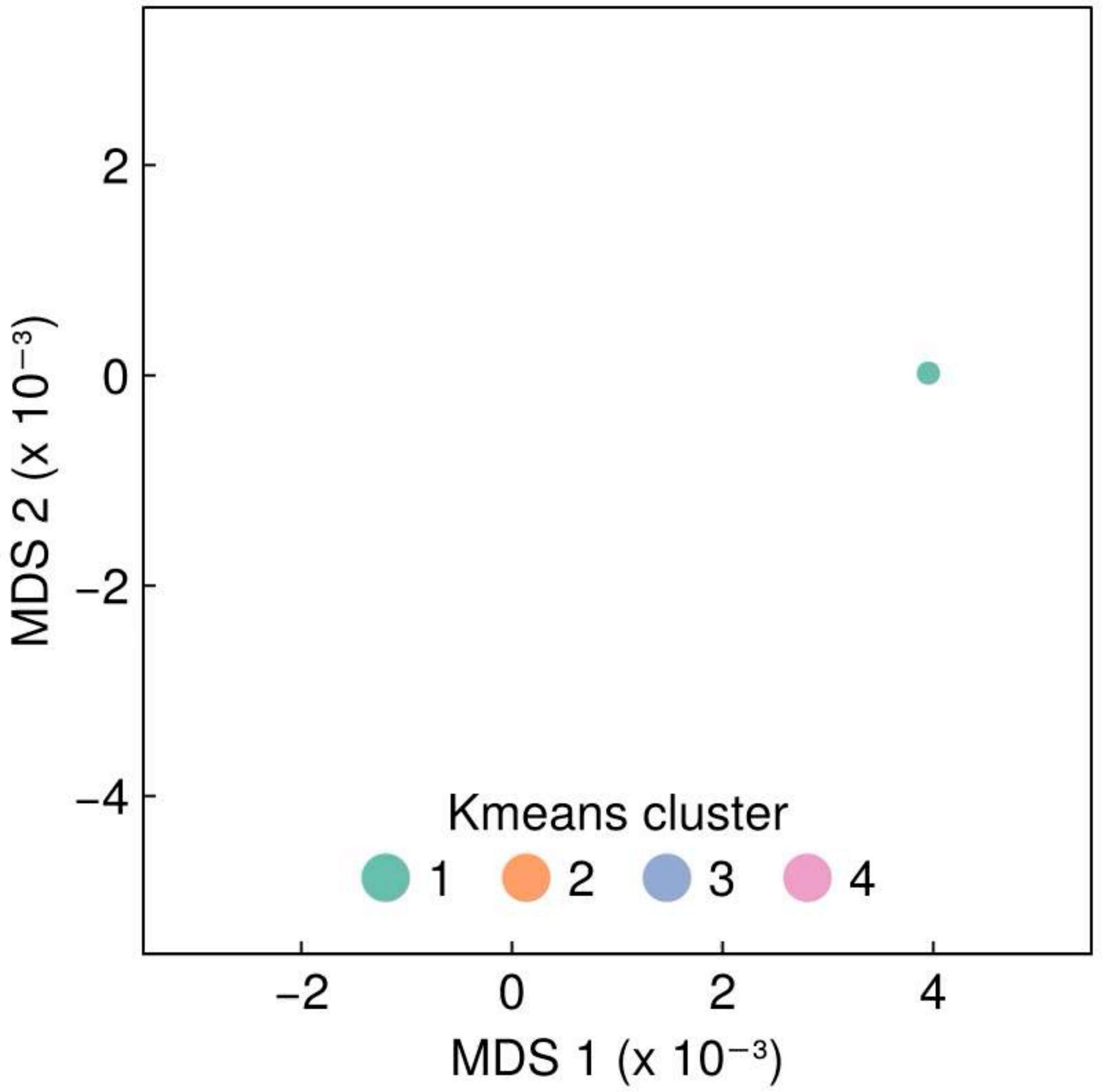
Morlet

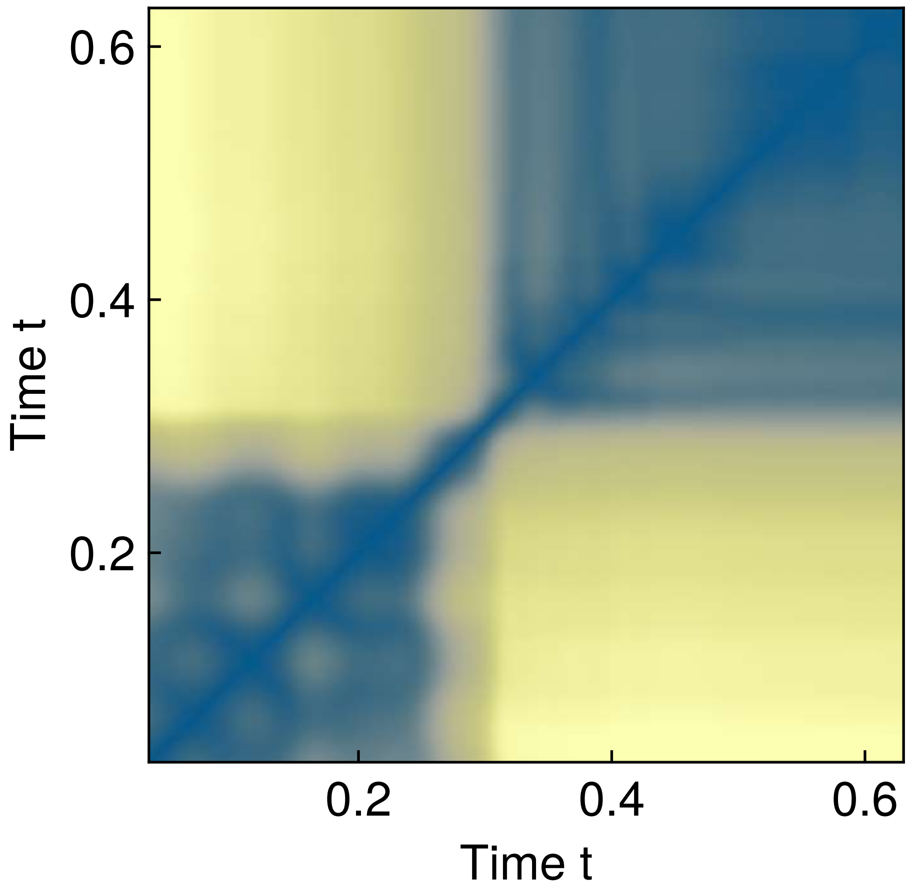






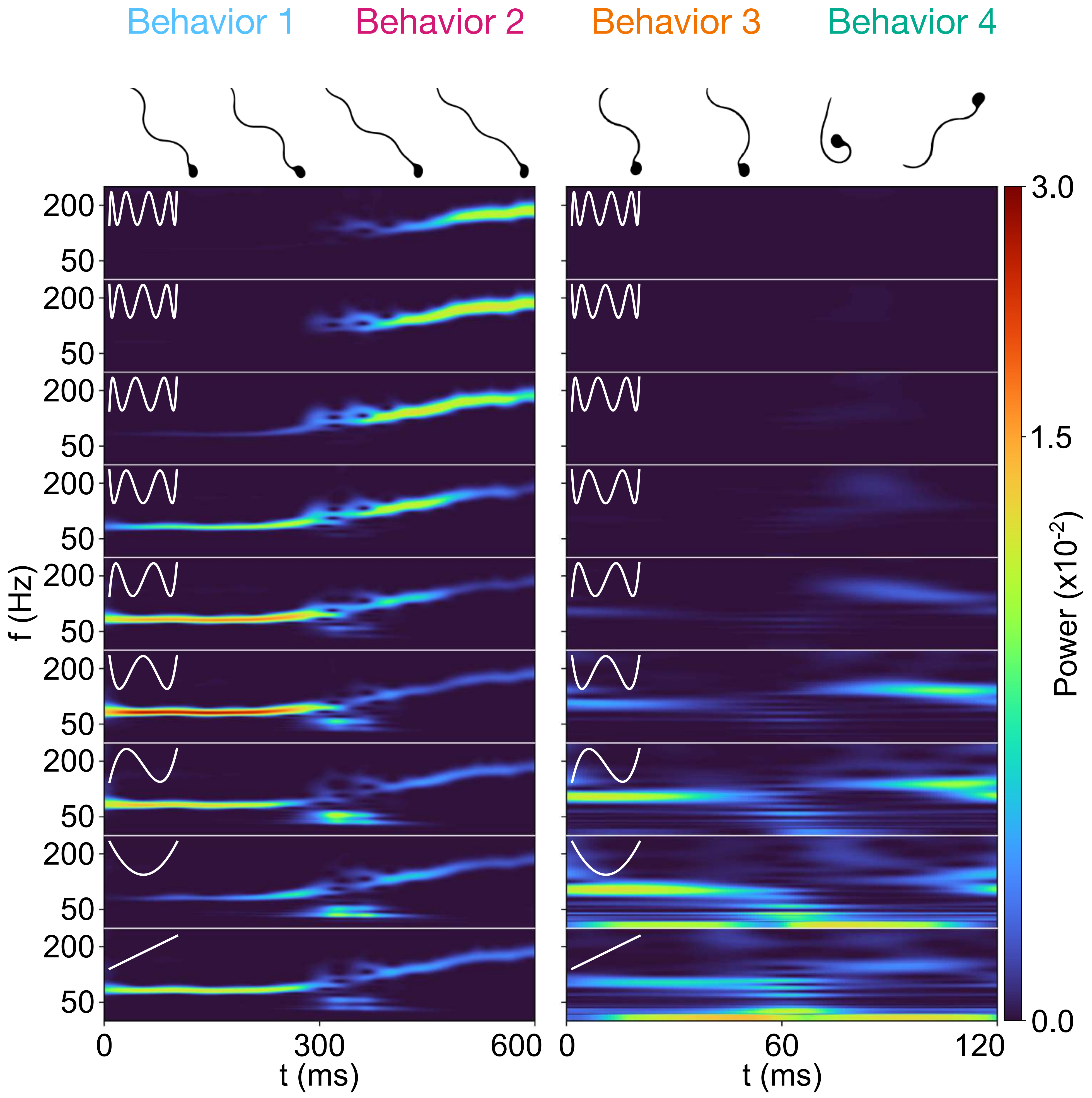


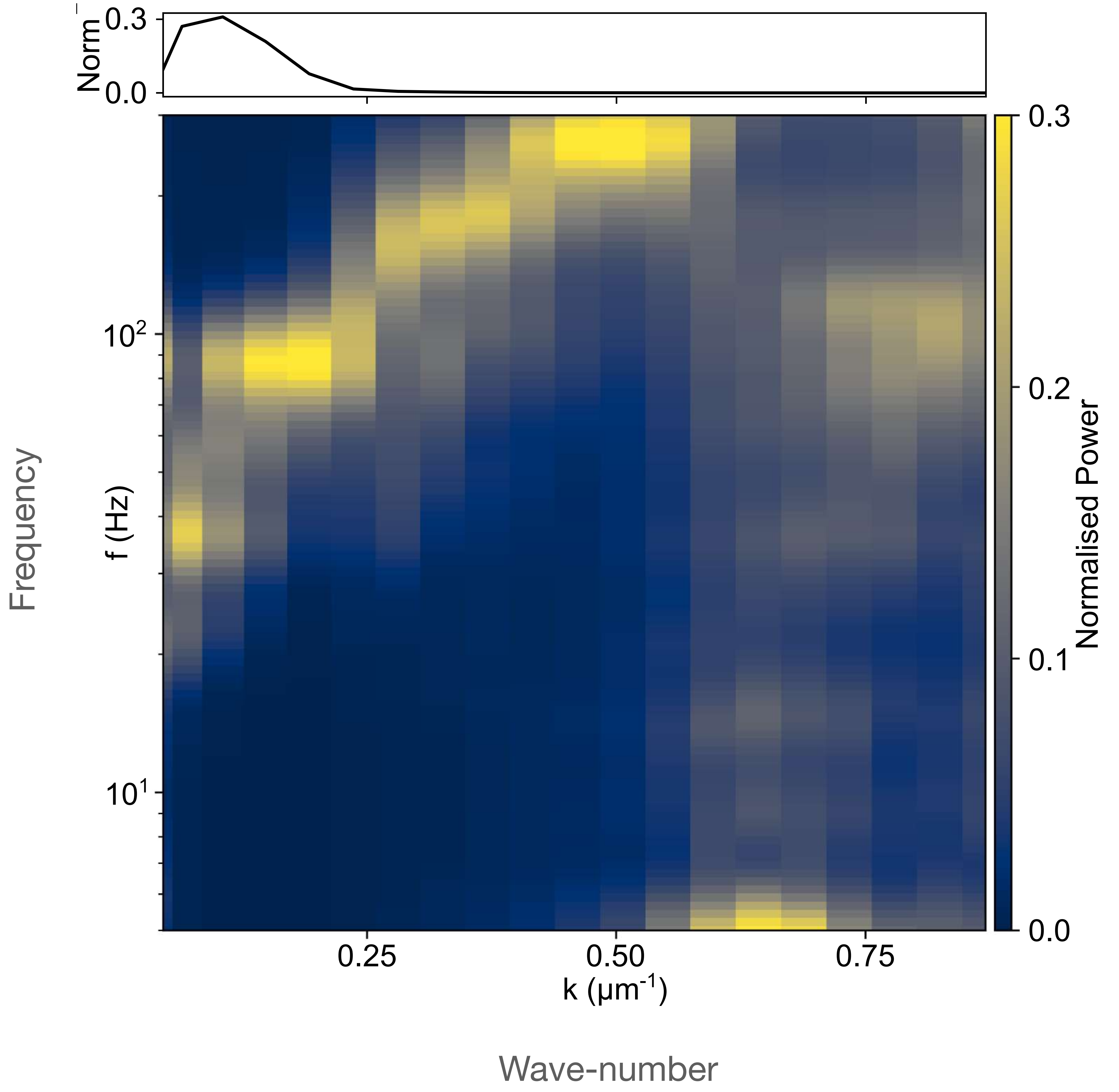




Instantaneous wavelet state
encodes
instantaneous dynamical behavior

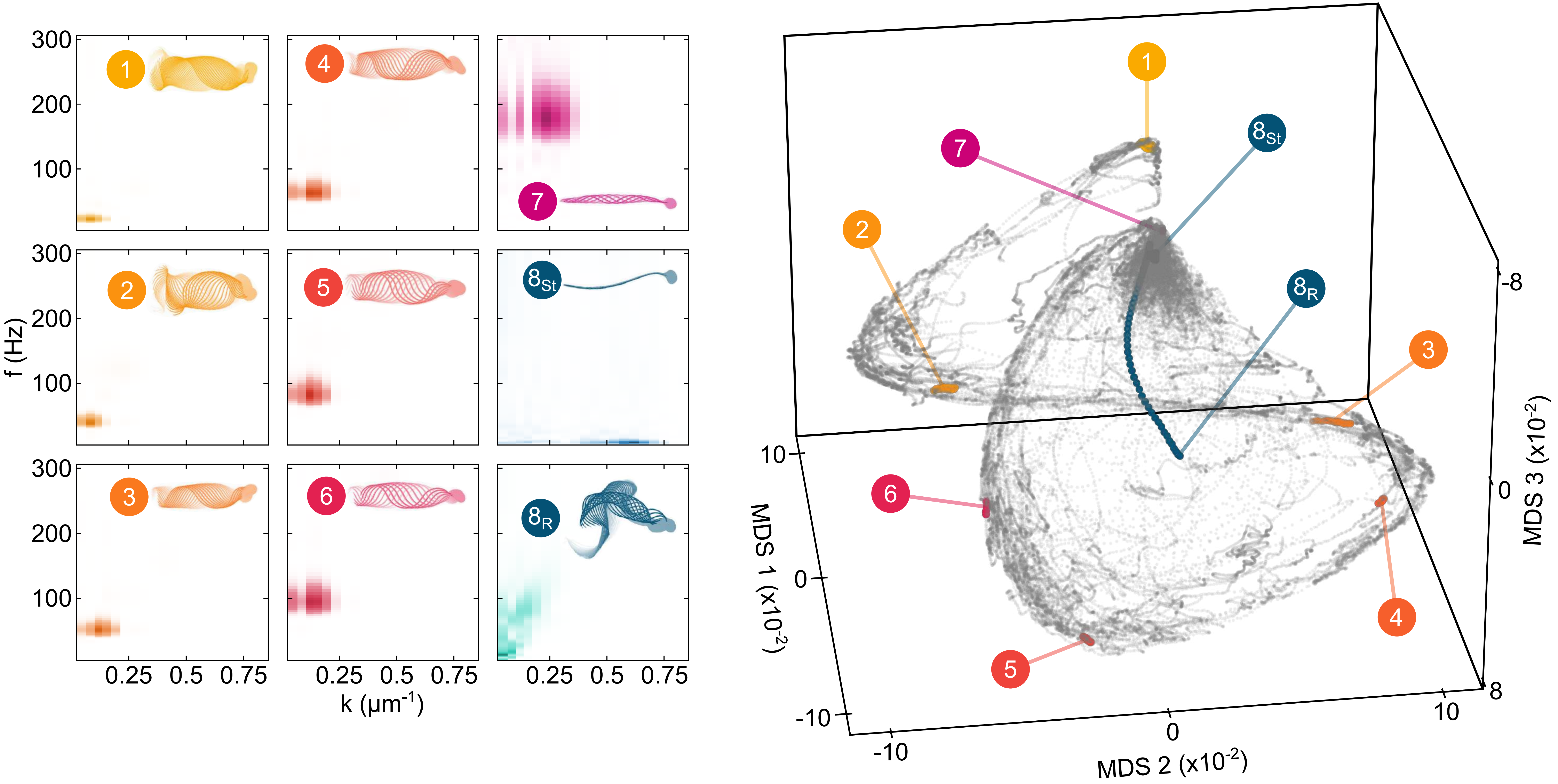
Dispersion relation





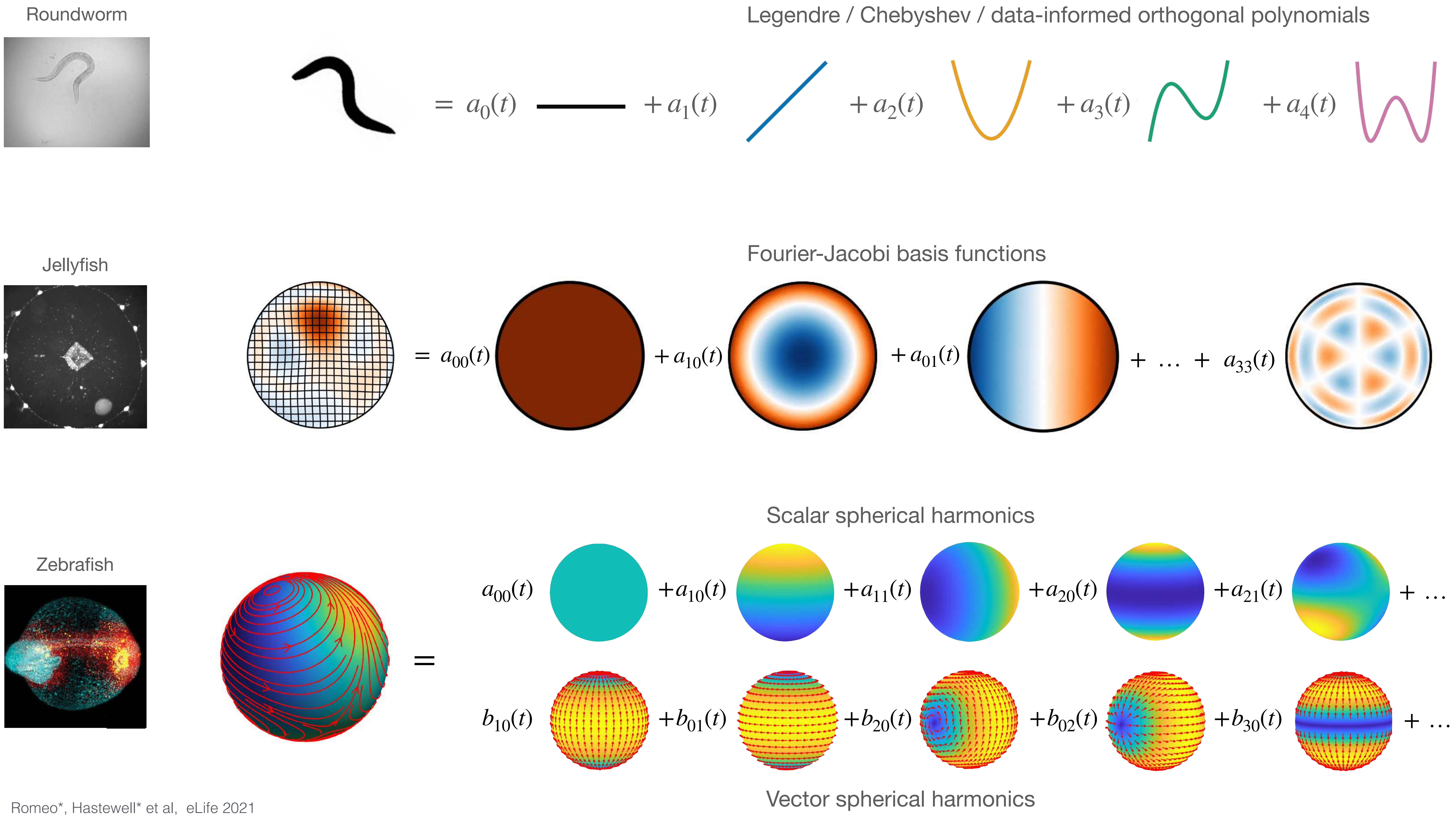
Behavioral manifold

Wave-modes activated in a specific behavioral state



Low-dimensional embedding (MDS based on distances between stacked wavelet-vectors)

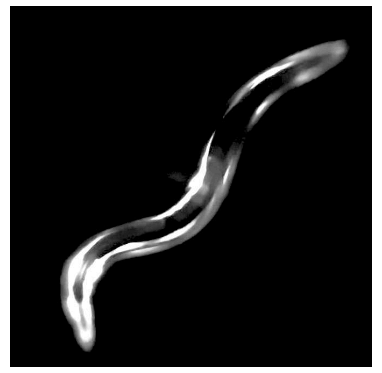
Generalizes across systems & scales



Romeo*, Hastewell* et al, eLife 2021

Can we learn predictive models of biophysical dynamics in spectral space ?

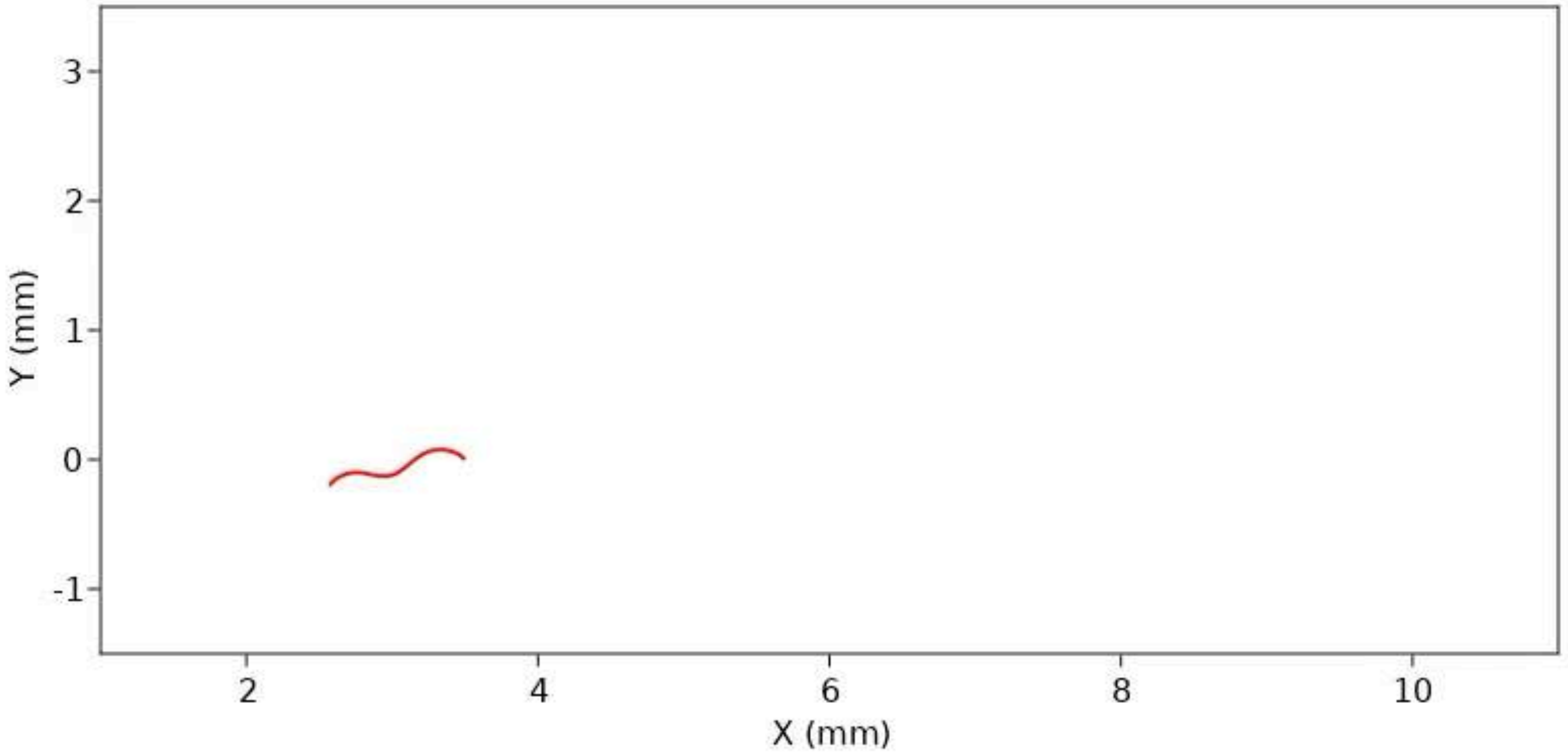
Experimental motivation

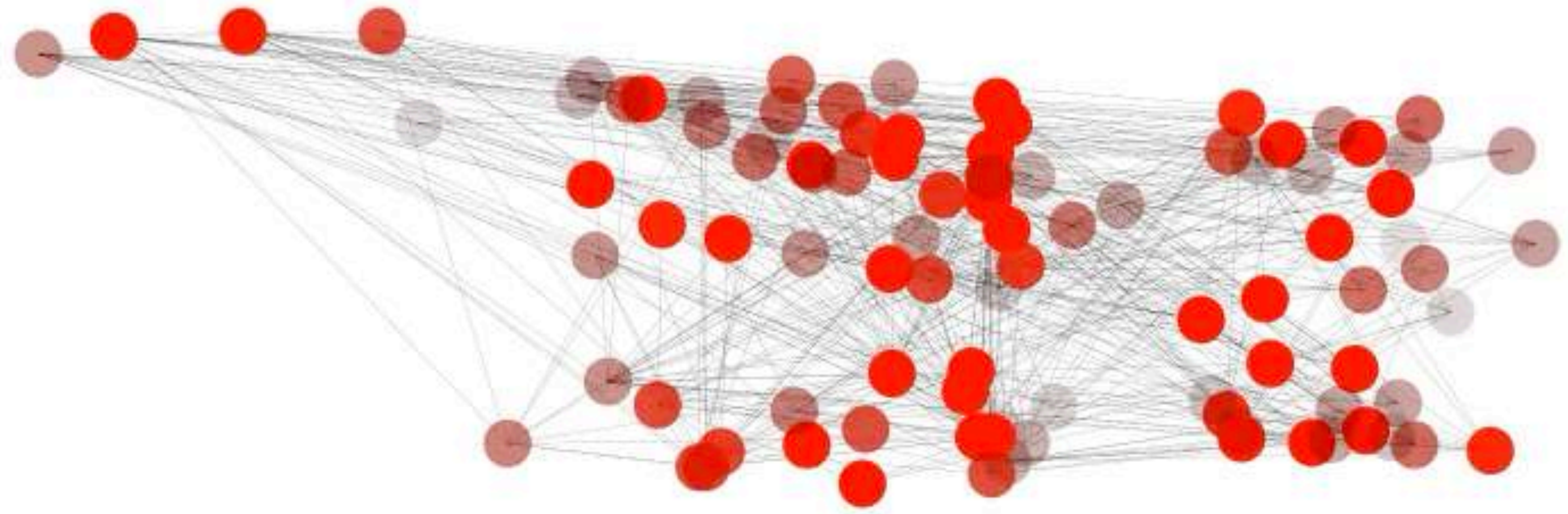


C elegans

physical locomotion dynamics ⇄ spatiotemporal neuron activity

Phys Rev Lett 2023





Steven Flavell

MIT | brain+cognitive sciences

Neuromechanics

Cohen & JD, preprint 2025

Model learning : increment-based

Given measured time series data $x(t_1), x(t_2), \ldots$ find 'simple' dynamical model

Deterministic

$$\dot{x} = f(x \,|\, p)$$

$$0 = dx - f(x \,|\, p)dt$$

$$\min_p |\, dx - f(x \,|\, p)dt \,|$$

constraint optimization
(sparsity, symmetries, …)

$$dx = x(t_{i+1}) - x(t_i)$$

$$dt = t_{i+1} - t_i$$

Model learning : increment-based

Given measured time series data $x(t_1), x(t_2), \ldots$ find 'simple' dynamical model

Deterministic

$$\dot{x} = f(x\,|\,p)$$

$$0 = dx - f(x\,|\,p)dt$$

$$\min_p |\,dx - f(x\,|\,p)dt\,|$$

constraint optimization
(sparsity, symmetries, …)

Stochastic

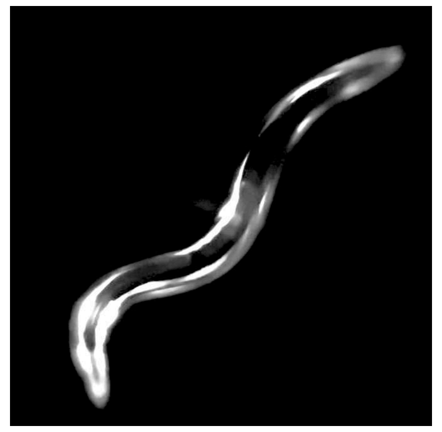
$$dx(t) = f(x\,|\,p)dt + g(x\,|\,p)dB(t)$$

$$dB = g^{-1}(dx - f\,dt)$$

$$\mathbb{P}[dB] = \mathbb{P}[g^{-1}(dx - fdt)]$$

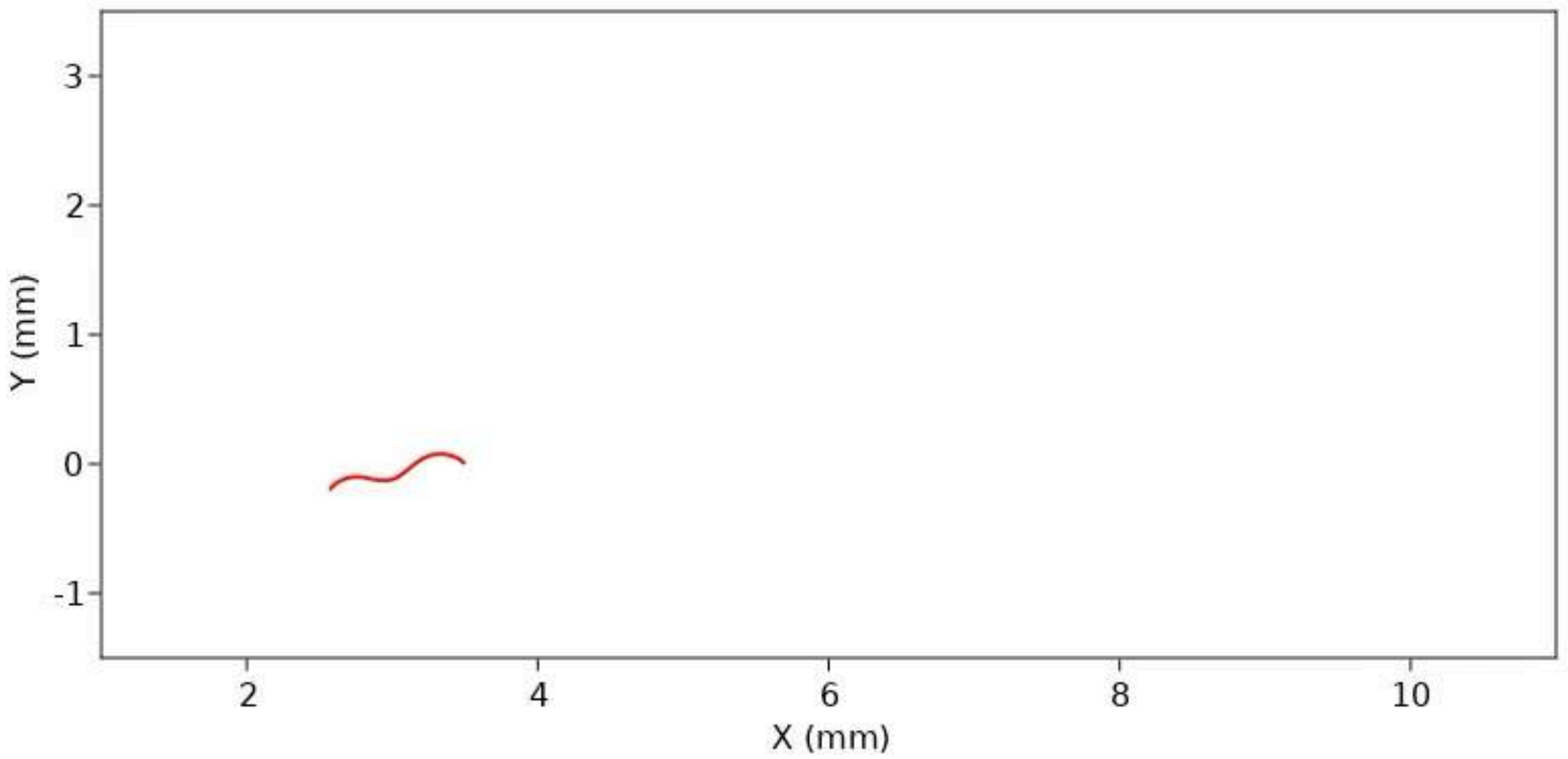
maximum likelihood estimation of p

Learning linear models for undulatory locomotion dynamics



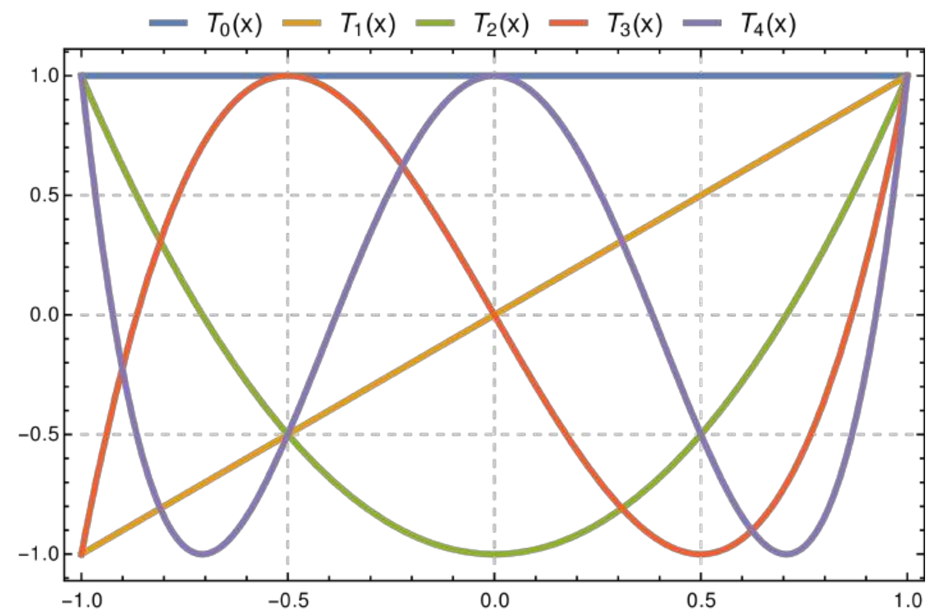
Flavell lab
(MIT BCS)

Experimental worm trajectories



$$\begin{bmatrix} x(s,t) \\ y(s,t) \end{bmatrix} = \sum_{k=0}^{n} T_k(s) \begin{bmatrix} \hat{x}_k(t) \\ \hat{y}_k(t) \end{bmatrix}$$

$$\begin{bmatrix} \hat{x}_k(t) \\ \hat{y}_k(t) \end{bmatrix} = \frac{\gamma_n}{\pi} \int_{-1}^{1} \mathrm{d}s \; w(s) \; T_k(s) \begin{bmatrix} x(s,t) \\ y(s,t) \end{bmatrix}$$



Linear short-time dynamics

$$\frac{d}{dt} \begin{bmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{y}} \end{bmatrix} = \mathbf{M} \begin{bmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{y}} \end{bmatrix}$$

$$\boldsymbol{\psi}(t) \sim \begin{bmatrix} \hat{x}_1 + i\hat{y}_1 \\ \vdots \\ \hat{x}_N + i\hat{y}_N \end{bmatrix}$$

Later: complex representation

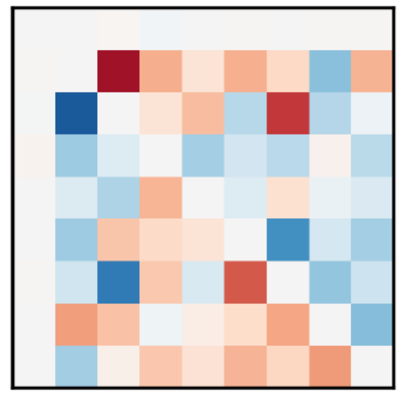
Cohen*, Hastewell* et al, PRL 2023

Symmetry & biophysical constraints

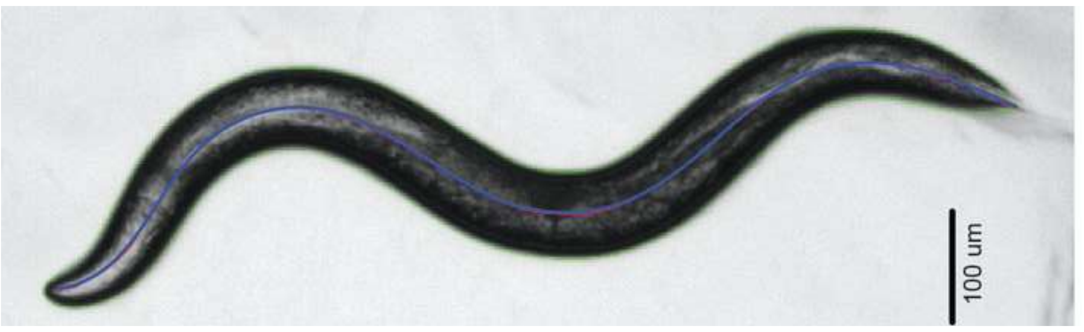
Constraints	Model (straight motion)	Degrees of freedom
None	$\frac{d}{dt}\begin{bmatrix}\hat{\mathbf{x}}\\ \hat{\mathbf{y}}\end{bmatrix} = \mathbf{M}^{(0)}\begin{bmatrix}\hat{\mathbf{x}}\\ \hat{\mathbf{y}}\end{bmatrix}$	$2N \times 2N$
Rotational Invariance	$\frac{d}{dt}\begin{bmatrix}\hat{\mathbf{x}}\\ \hat{\mathbf{y}}\end{bmatrix} = \begin{bmatrix}\mathbf{M}^{(1)} & 0\\ 0 & \mathbf{M}^{(1)}\end{bmatrix}\begin{bmatrix}\hat{\mathbf{x}}\\ \hat{\mathbf{y}}\end{bmatrix}$	$N \times N$
Length approx conserved	$\frac{d}{dt}\begin{bmatrix}\hat{\mathbf{x}}\\ \hat{\mathbf{y}}\end{bmatrix} = \begin{bmatrix}\mathbf{M}^{(2)}\mathbf{W} & 0\\ 0 & \mathbf{M}^{(2)}\mathbf{W}\end{bmatrix}\begin{bmatrix}\hat{\mathbf{x}}\\ \hat{\mathbf{y}}\end{bmatrix}$	$\frac{N \times (N-1)}{2}$

$$\mathbf{W}_{ij} = \int_{-1}^{1} ds \left(\frac{dT_i}{ds}\frac{dT_j}{ds}\right)$$

$$\mathbf{M}^{(2)} = -\mathbf{M}^{(2)^\top}$$



Flavell lab (MIT Picower)



Planar curve

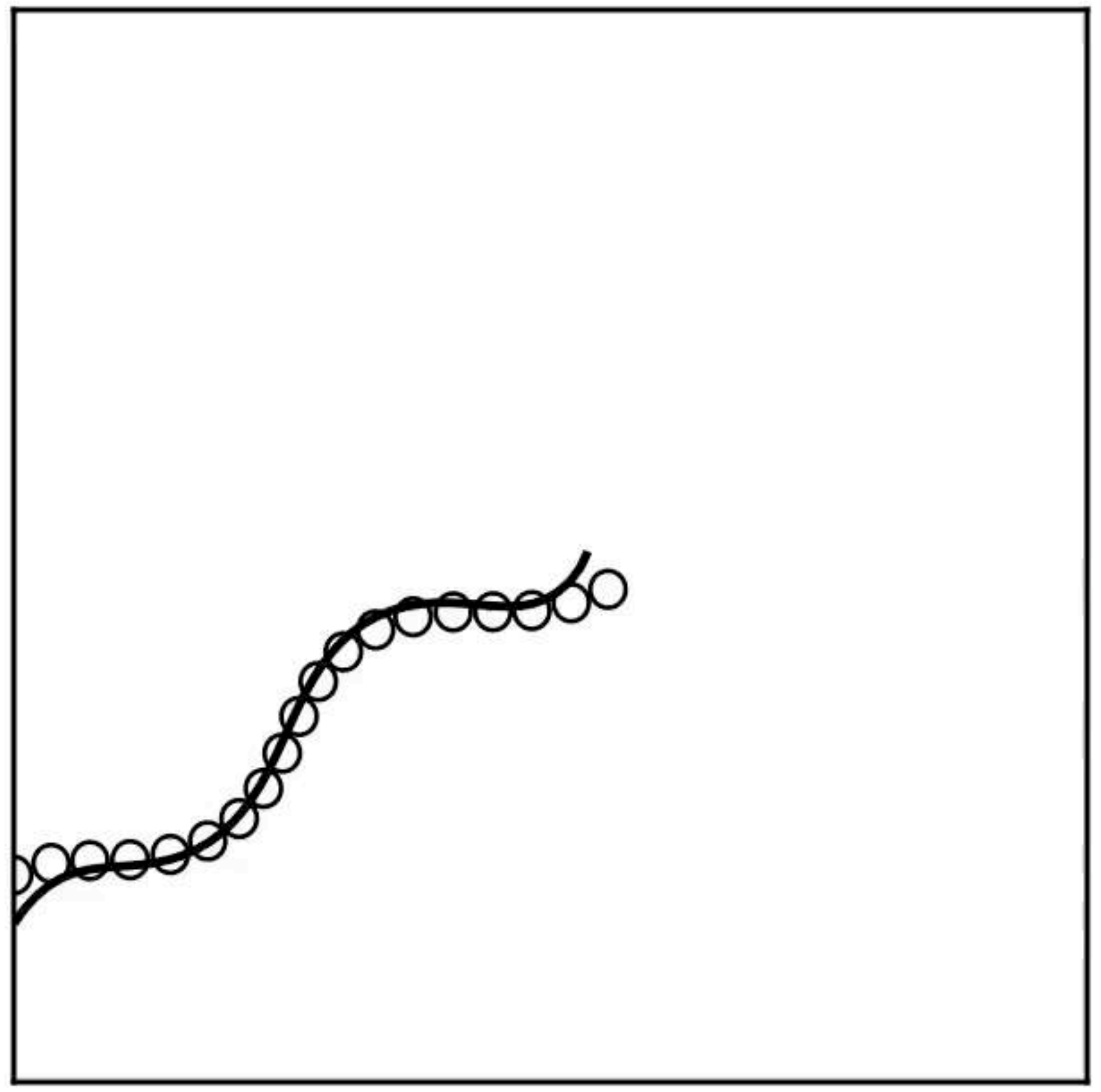
$$x(s,t) + iy(s,t) \Rightarrow$$

Center-of-mass

$$\psi_0(t) \sim \hat{x}_0 + i\hat{y}_0$$

Shape modes

$$\psi(t) \sim \begin{bmatrix} \hat{x}_1 + i\hat{y}_1 \\ \vdots \\ \hat{x}_N + i\hat{y}_N \end{bmatrix}$$



$$\psi^\dagger \psi = 1$$

'Length' constraint

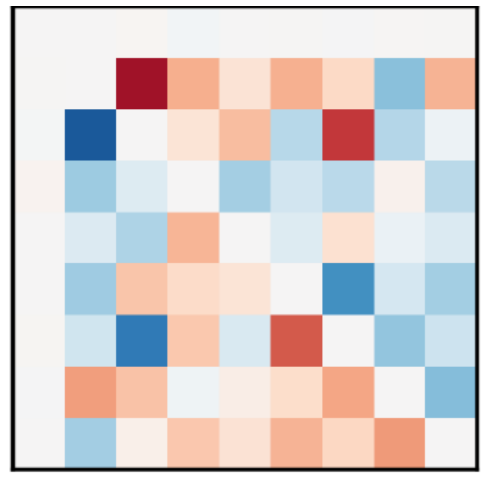
$$i\dot{\psi} = H\psi$$

Shape mode dynamics

$$\dot{\psi}_0 = H_0\psi$$

Propulsion dynamics

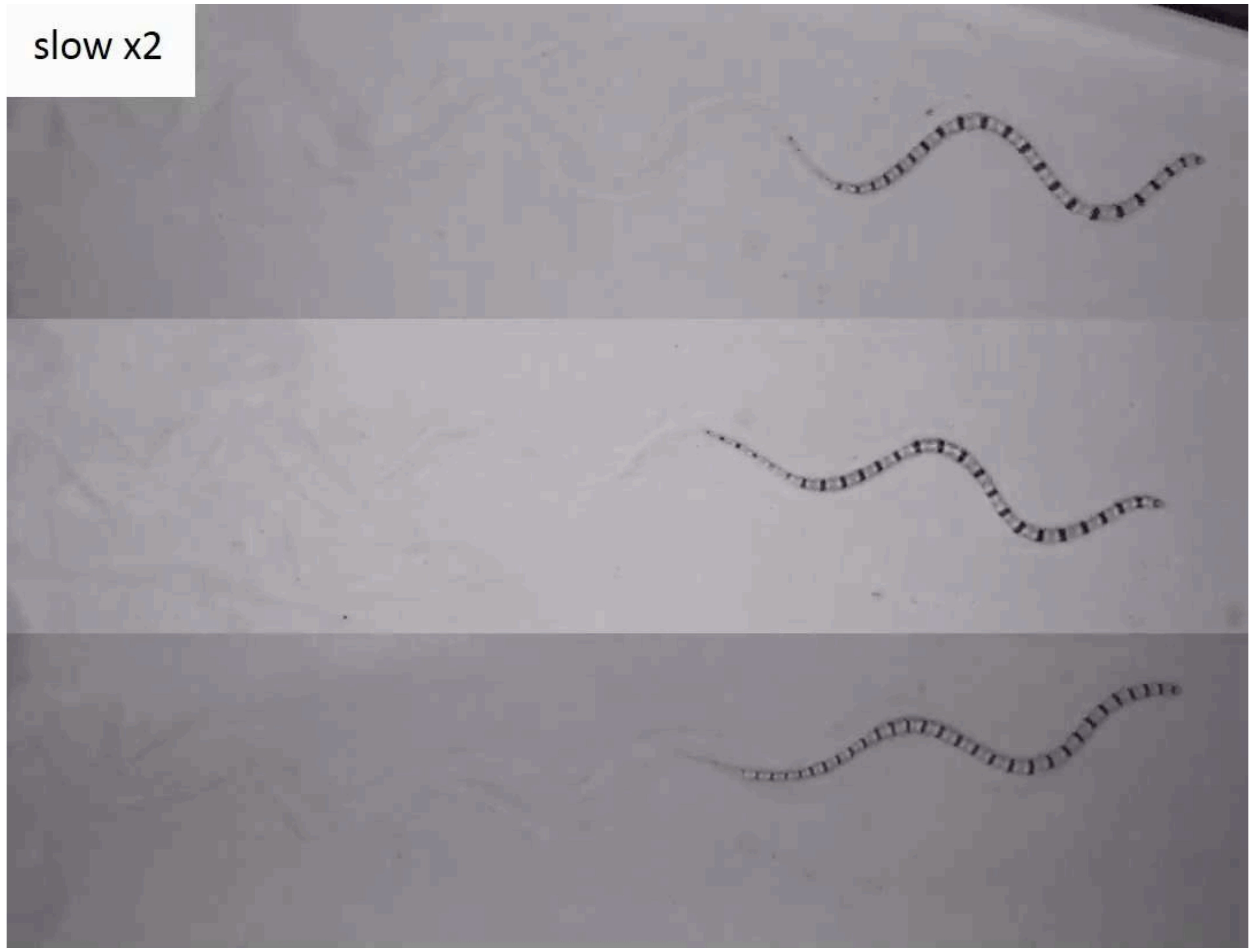
H



$$H = \cancel{S} + iA$$

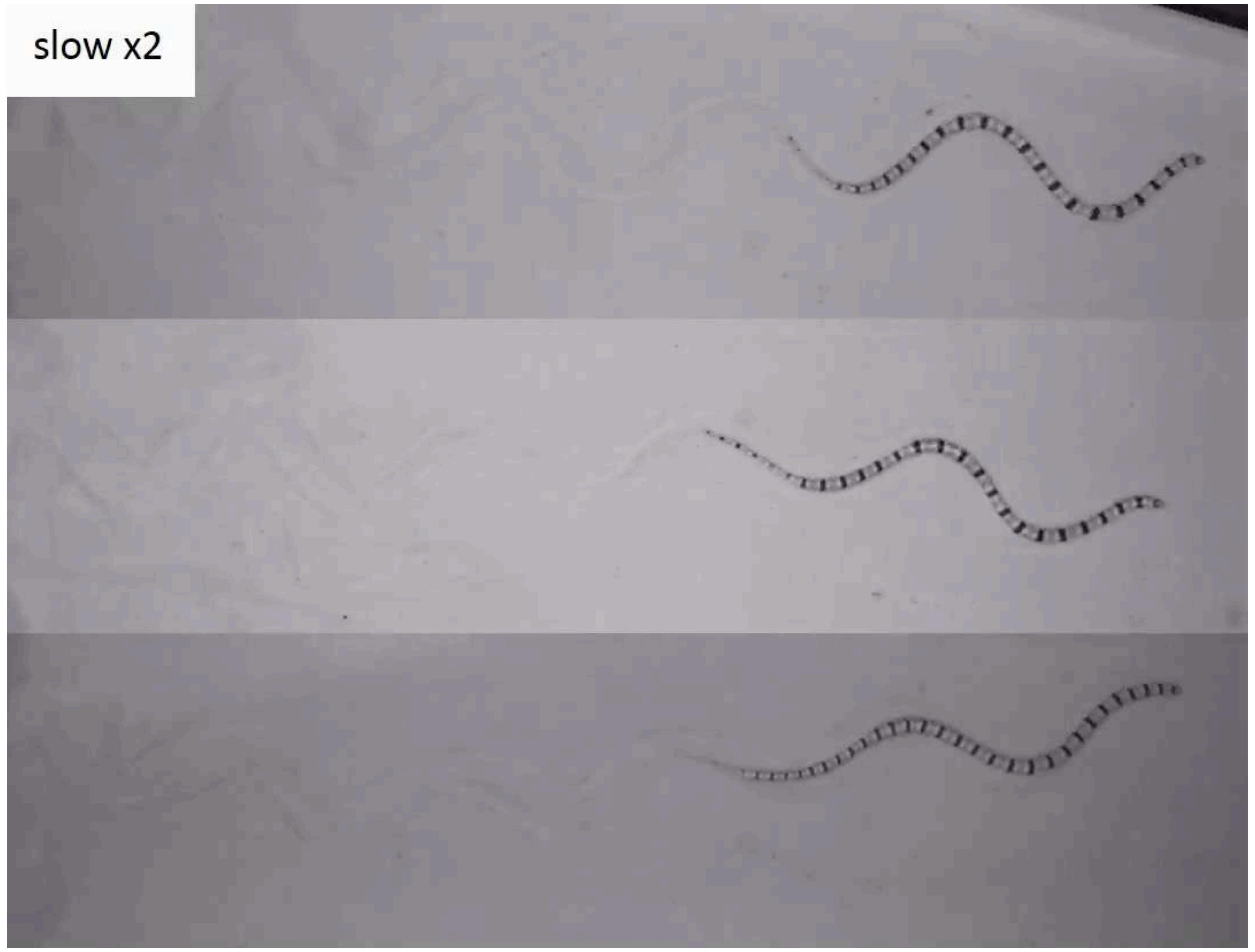
Straight motion

Desert snake



Goldman Lab (Georgia Tech), PNAS 2019

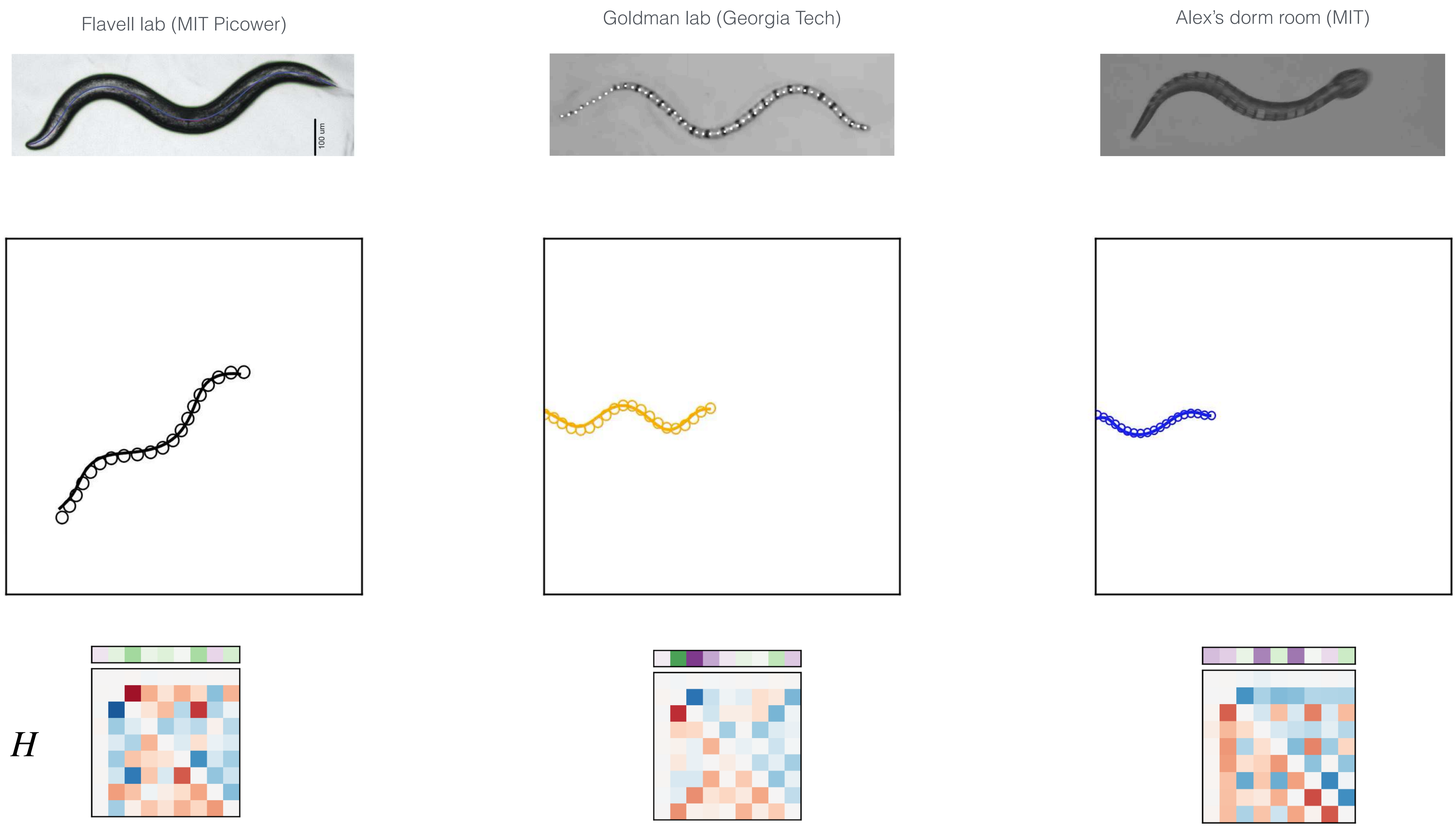
Desert snake



Goldman Lab (Georgia Tech), PNAS 2019

Toy snake

Cohen*, Hastewell* et al, PRL 2023

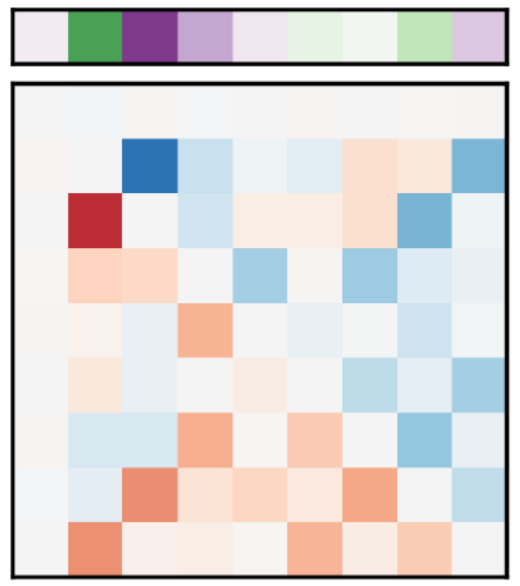

Flavell lab (MIT Picower)
Goldman lab (Georgia Tech)
Alex's dorm room (MIT)
100 um
H

Schrödinger dynamics of undulatory locomotion

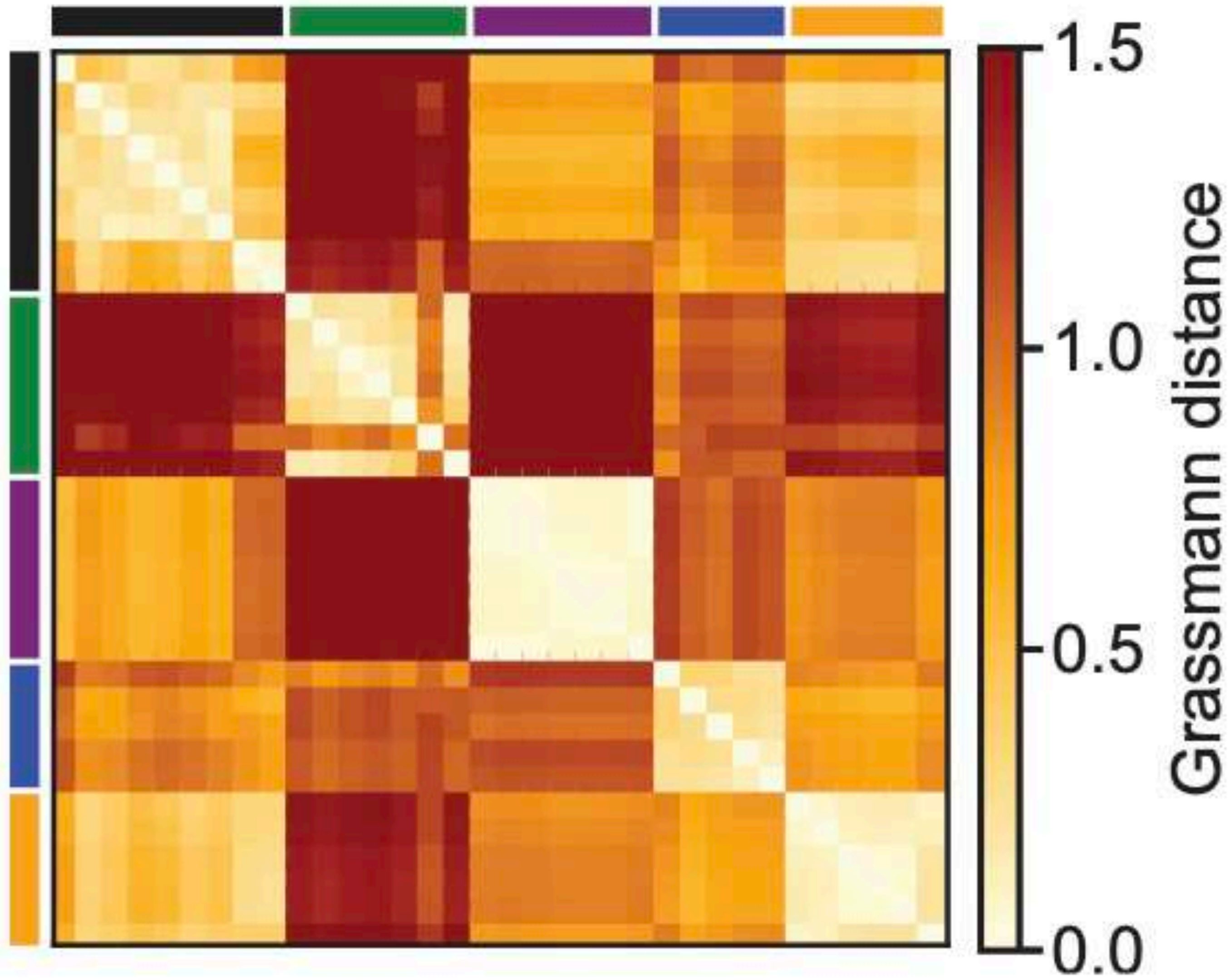
$$\begin{bmatrix} x(s,t) \\ y(s,t) \end{bmatrix} = \sum_{k=0}^{n} T_k(s) \begin{bmatrix} \hat{x}_k(t) \\ \hat{y}_k(t) \end{bmatrix}$$

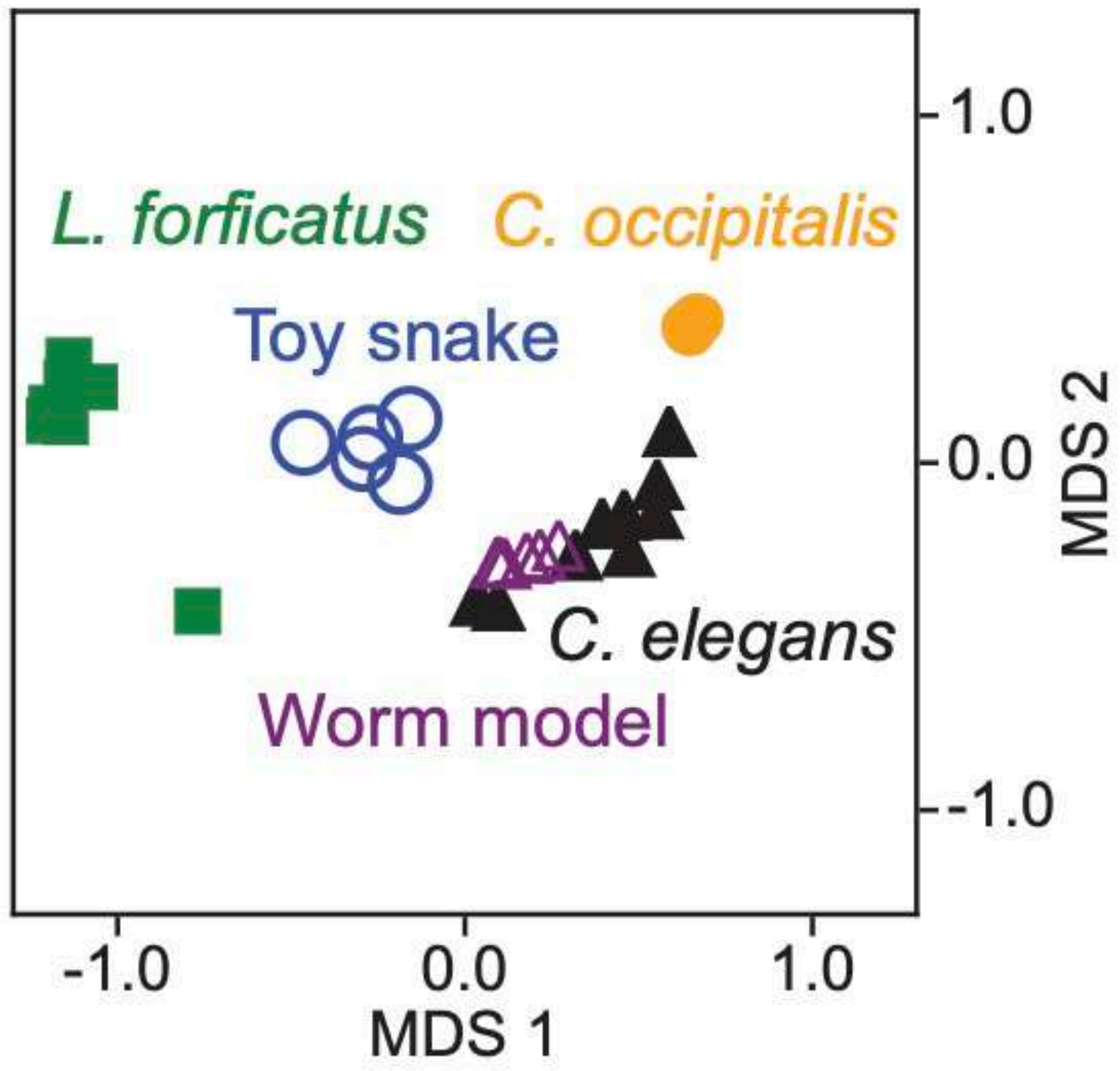
$$\dot{\psi}_0 = H_0 \psi$$

$$i\dot{\psi} = H\psi$$



$$1 = \psi^\dagger \psi$$

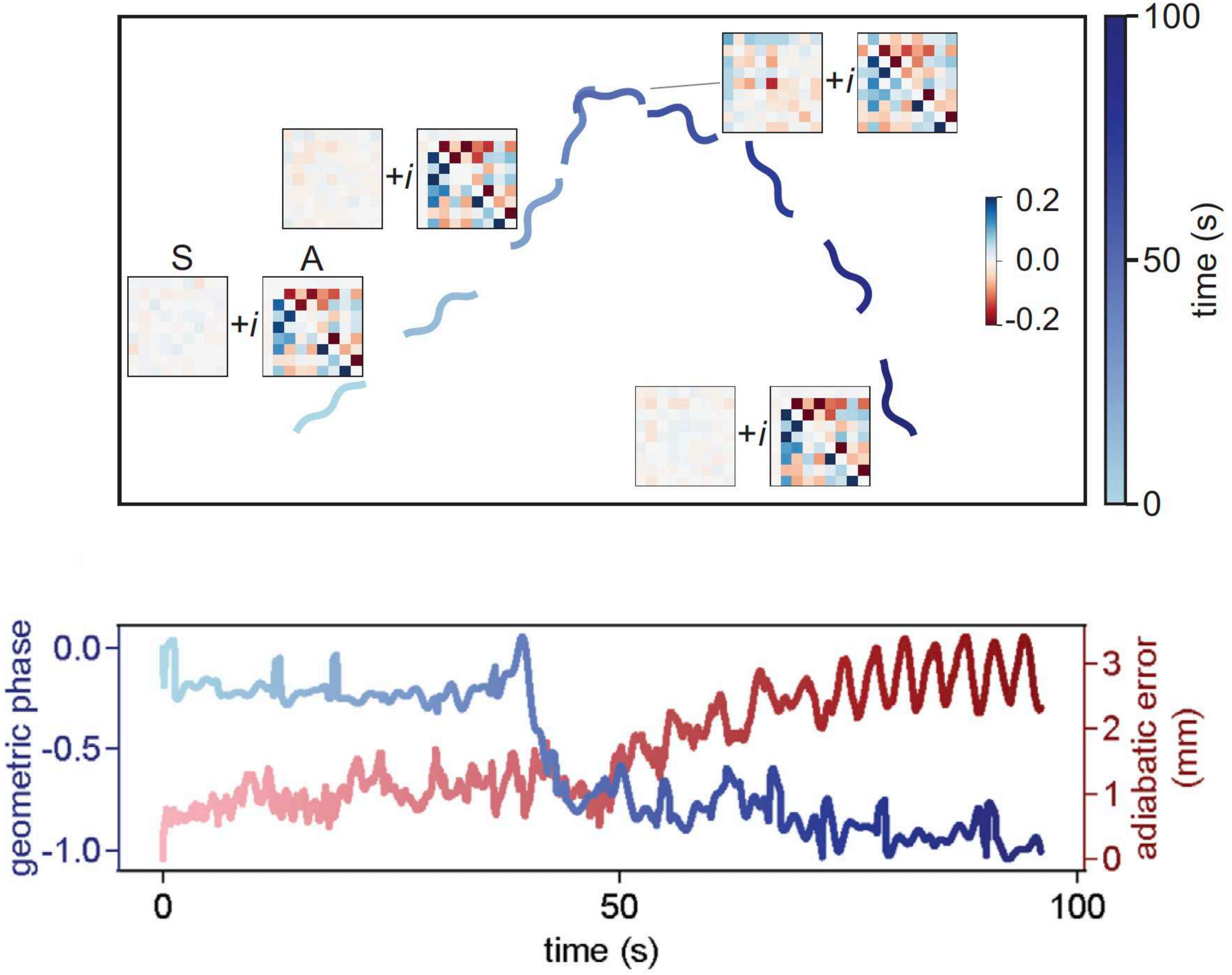




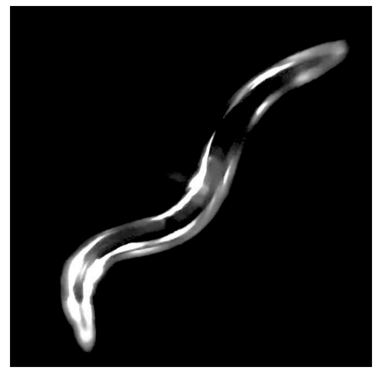
behavioral state ⇔ low-rank Hamiltonian ⇒ interspecies comparison

Cohen*, Hastewell* et al, Phys Rev Lett 2023

Time-dependent Hamiltonian and non-adiabatic turning



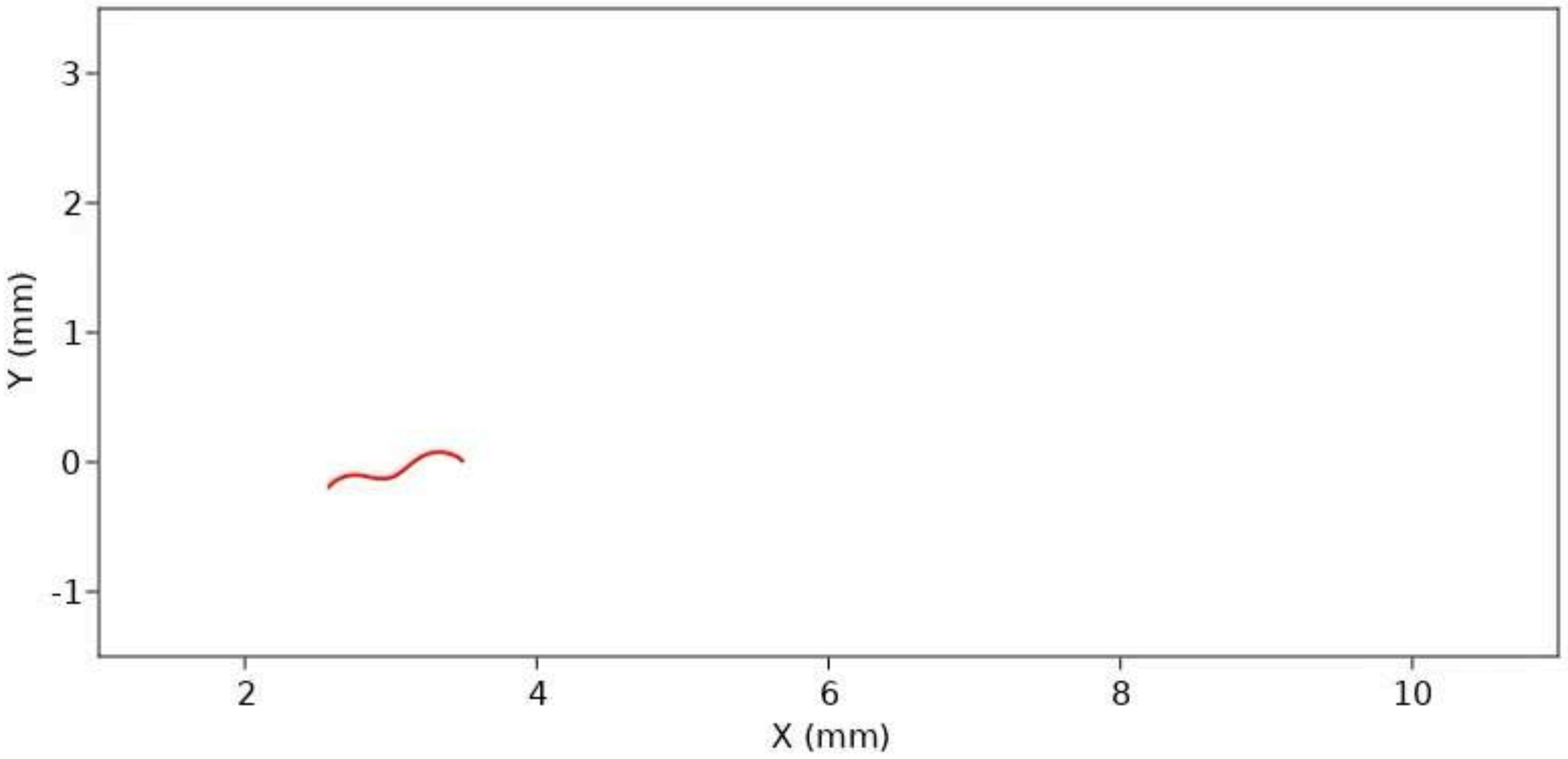
Outlook : Nonlinear model inference

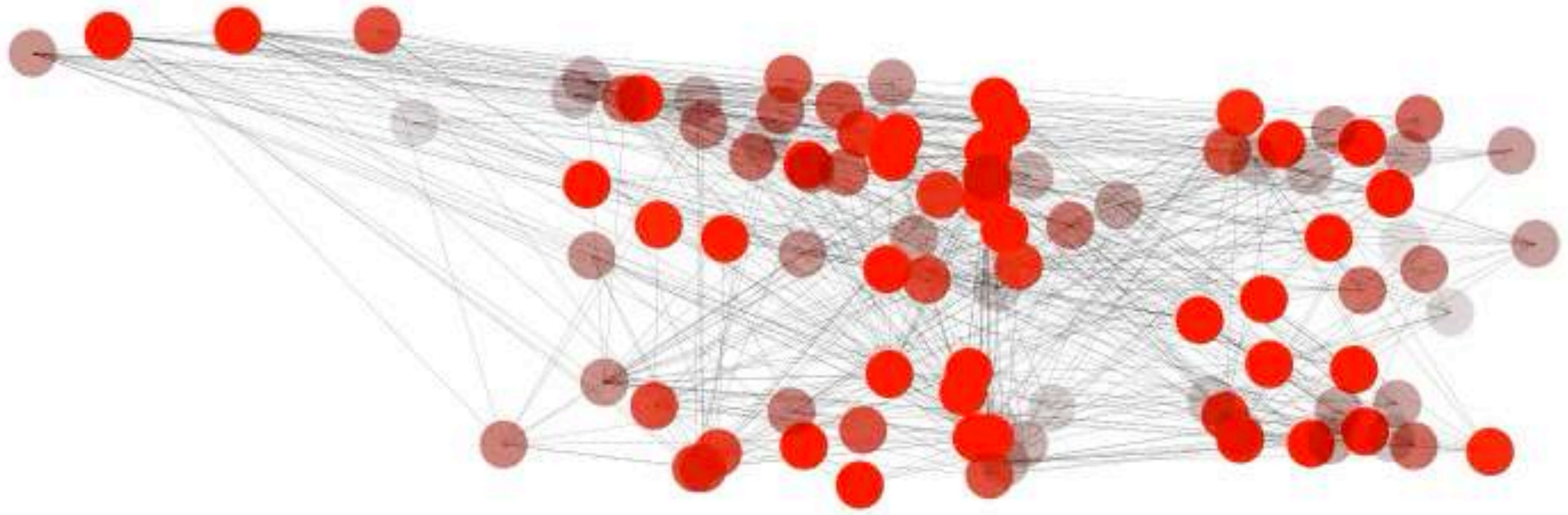


C elegans

Spatiotemporal neuron activity ⇄ physical dynamics

Phys Rev Lett 2023





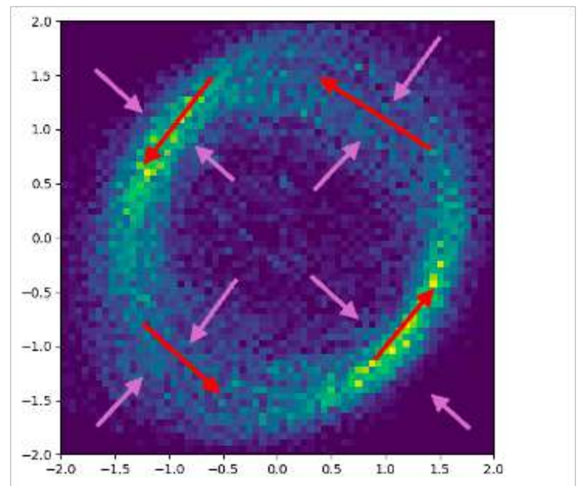
Steven Flavell

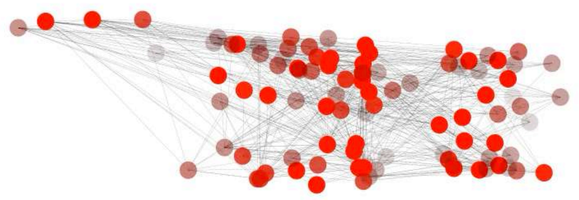
brain+cognitive sciences

Neuromechanics

Cohen et al, preprint 2025 (available on request)

Outlook : Nonlinear SDE inference

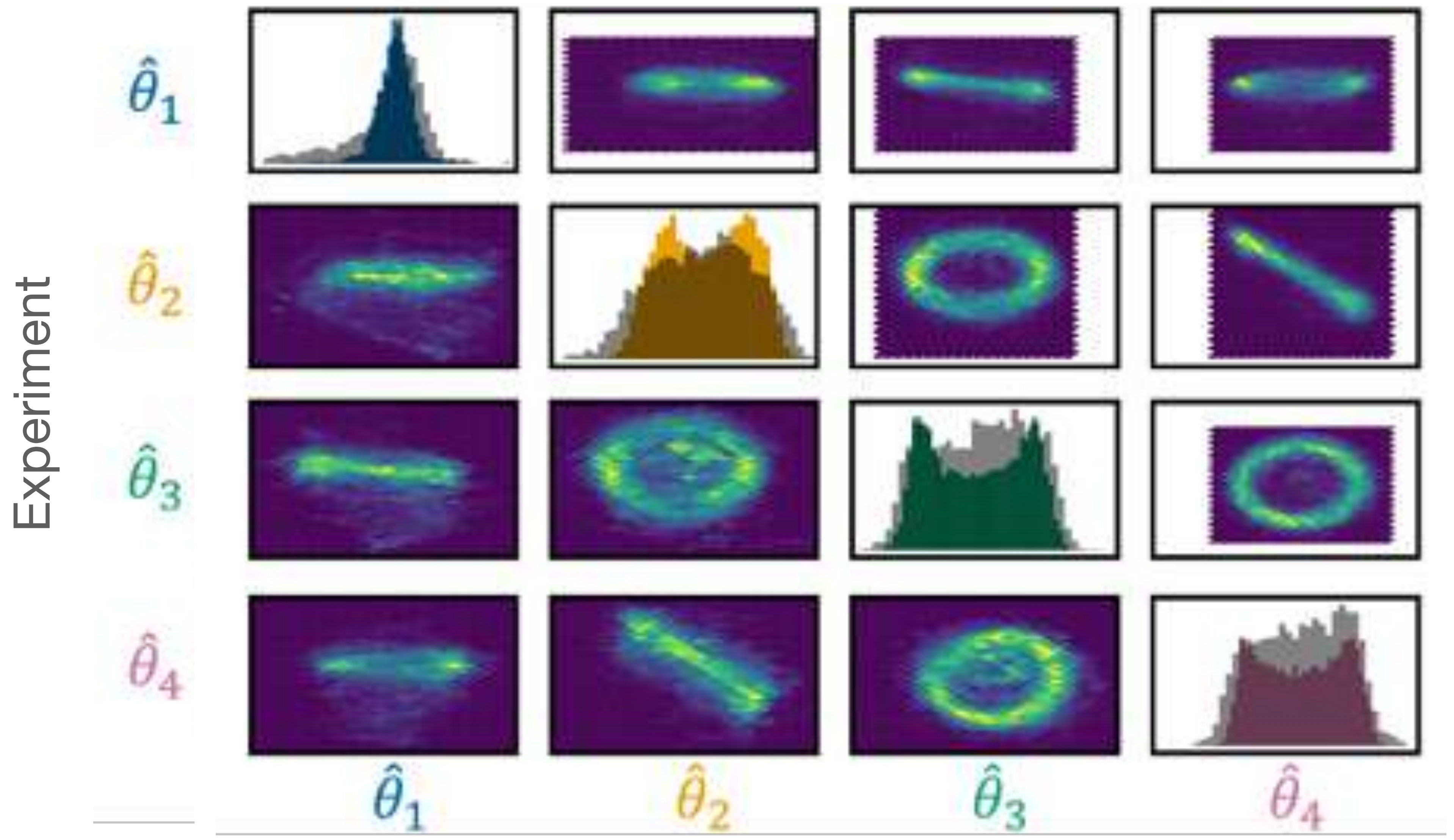




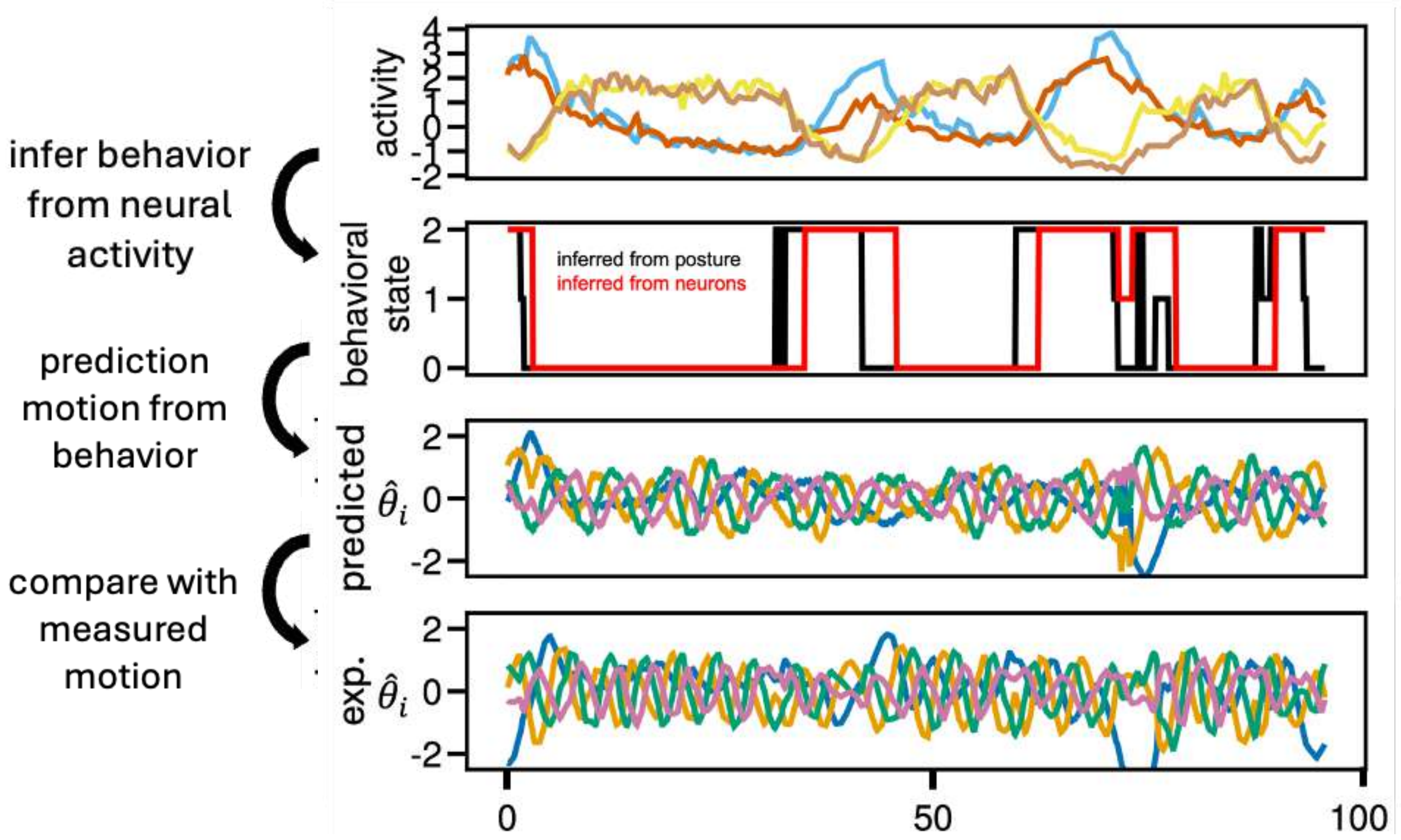
$$d\hat{\theta}_i = f_i(\hat{\boldsymbol{\theta}} \,|\, \mathbf{z}) + \sigma_{ij}(\mathbf{z})\, dB_j(t)$$

$$\mathbf{f} = \nabla V + \mathbf{c}$$

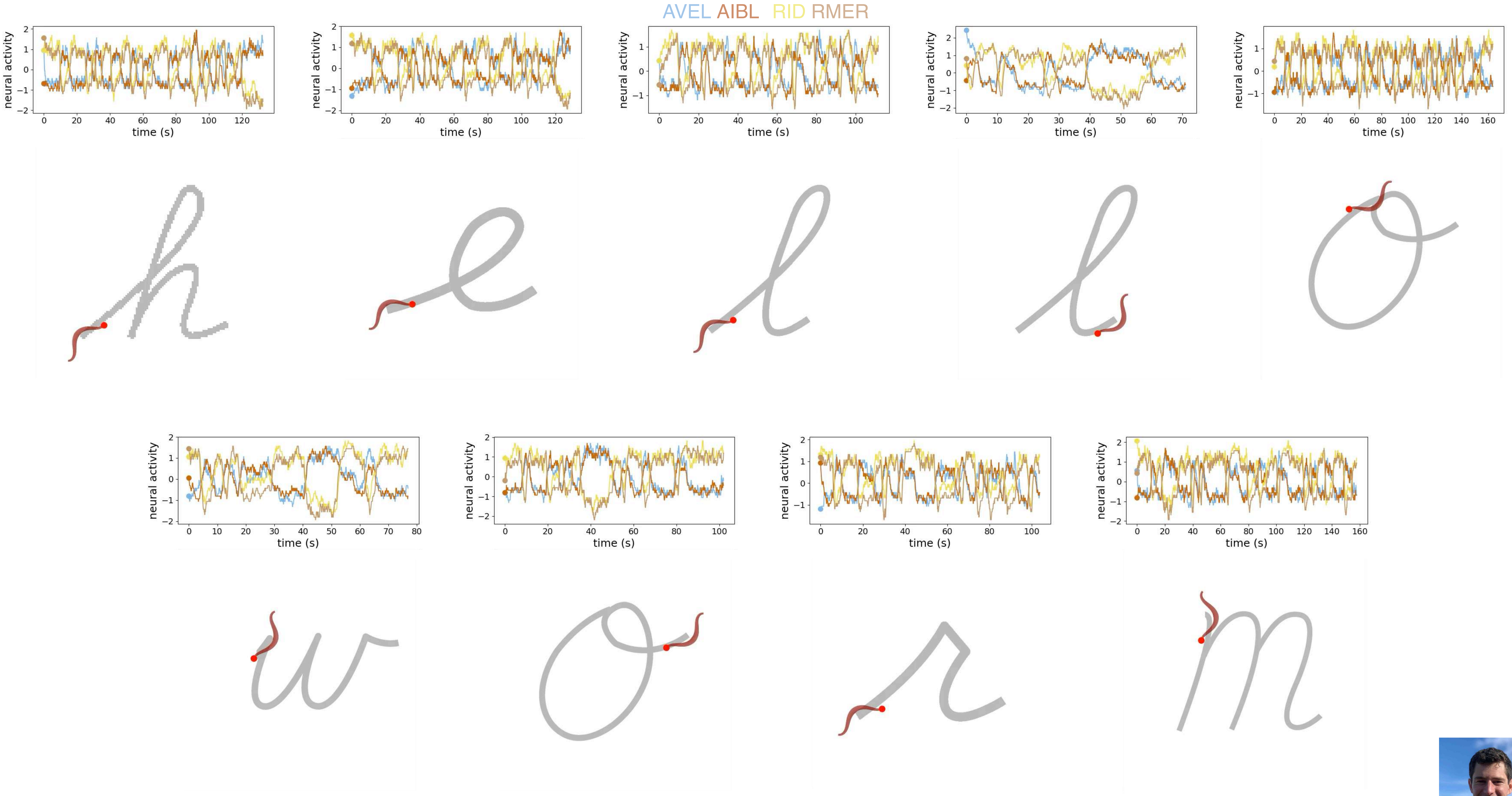
Nonlinear SDE model



Predicting shape-dynamics from neuron activity



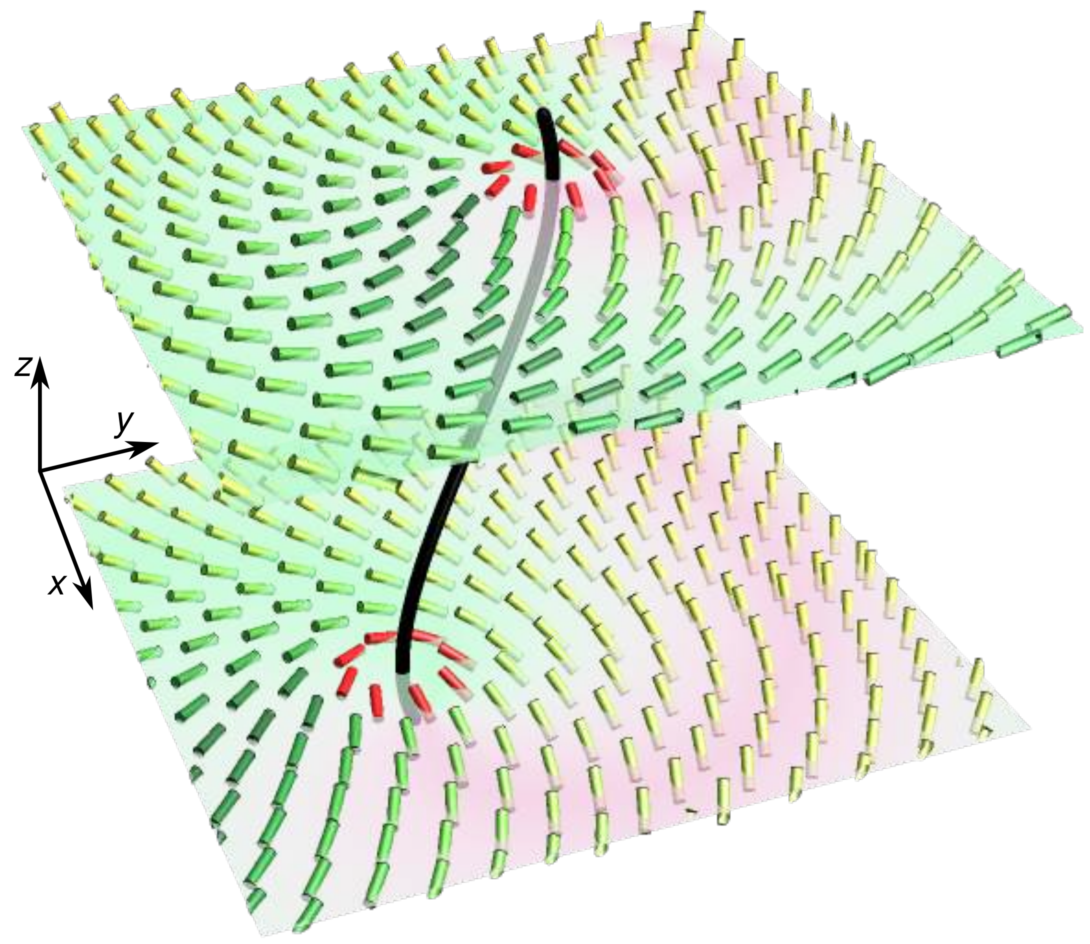
Cohen et al, preprint 2025 (available on request)



Cohen et al, preprint 2025 (available on request)

Part 2:

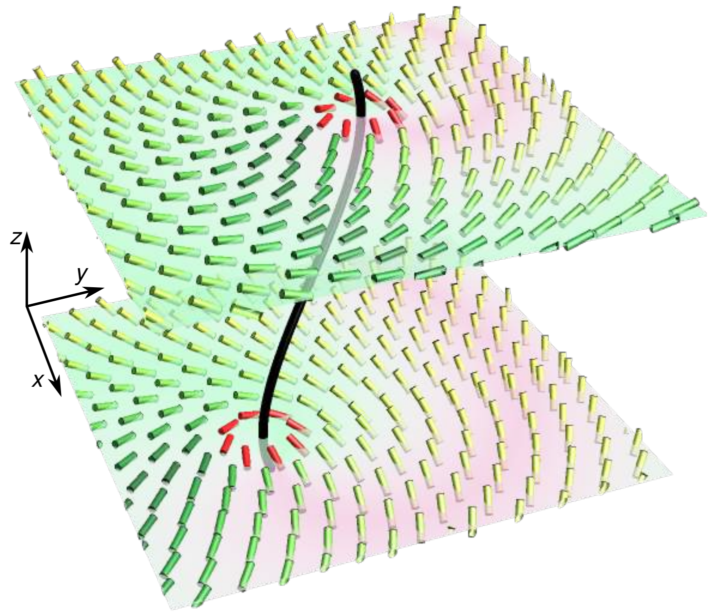
Topological defects & computation in liquid crystals



Science Advances 2022

PRX 2022

Broader motivation



Science Advances 2022

Ziga Kos
(MIT ⇒ Ljubljana)

Can we use topological defects as computational ‘bits’ ?

What types of computations can we perform ?

How can we manipulate defects to implement certain logical operations ?

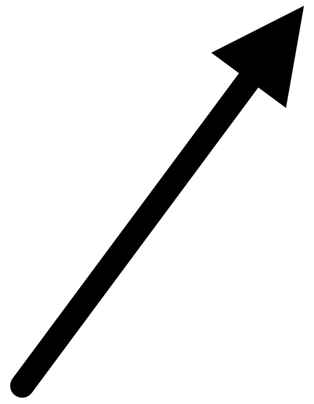
Borrow ideas from other fields by exploring analogies ?

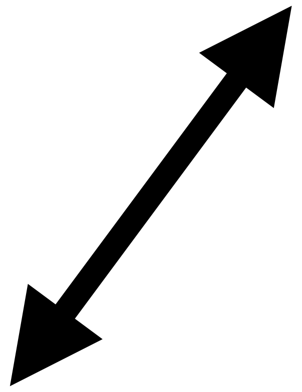
PRX 2022

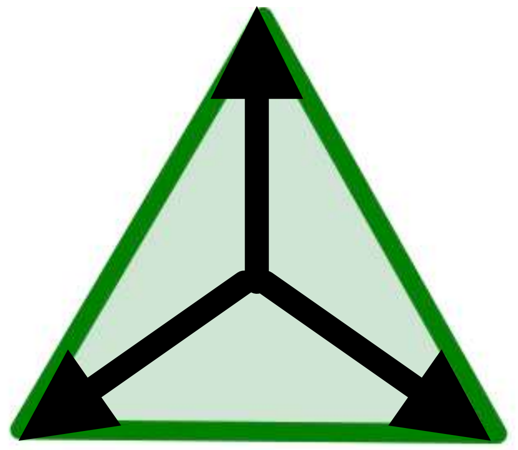
Alex Mietke
(MIT ⇒ Oxford)

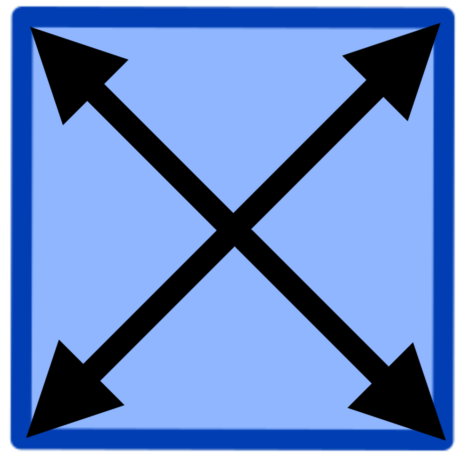
Dihedral particle symmetry

$k = 1$ $\quad$ $k = 2$ $\quad$ $k = 3$ $\quad$ $k = 4$



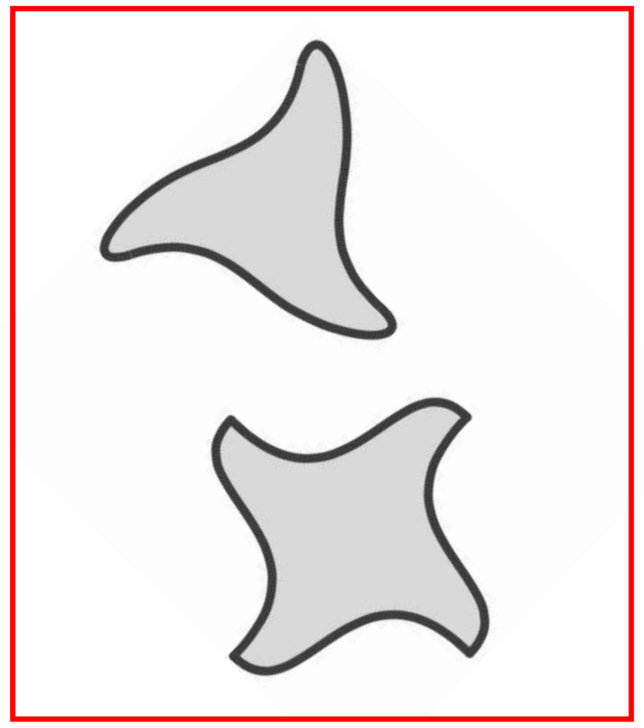






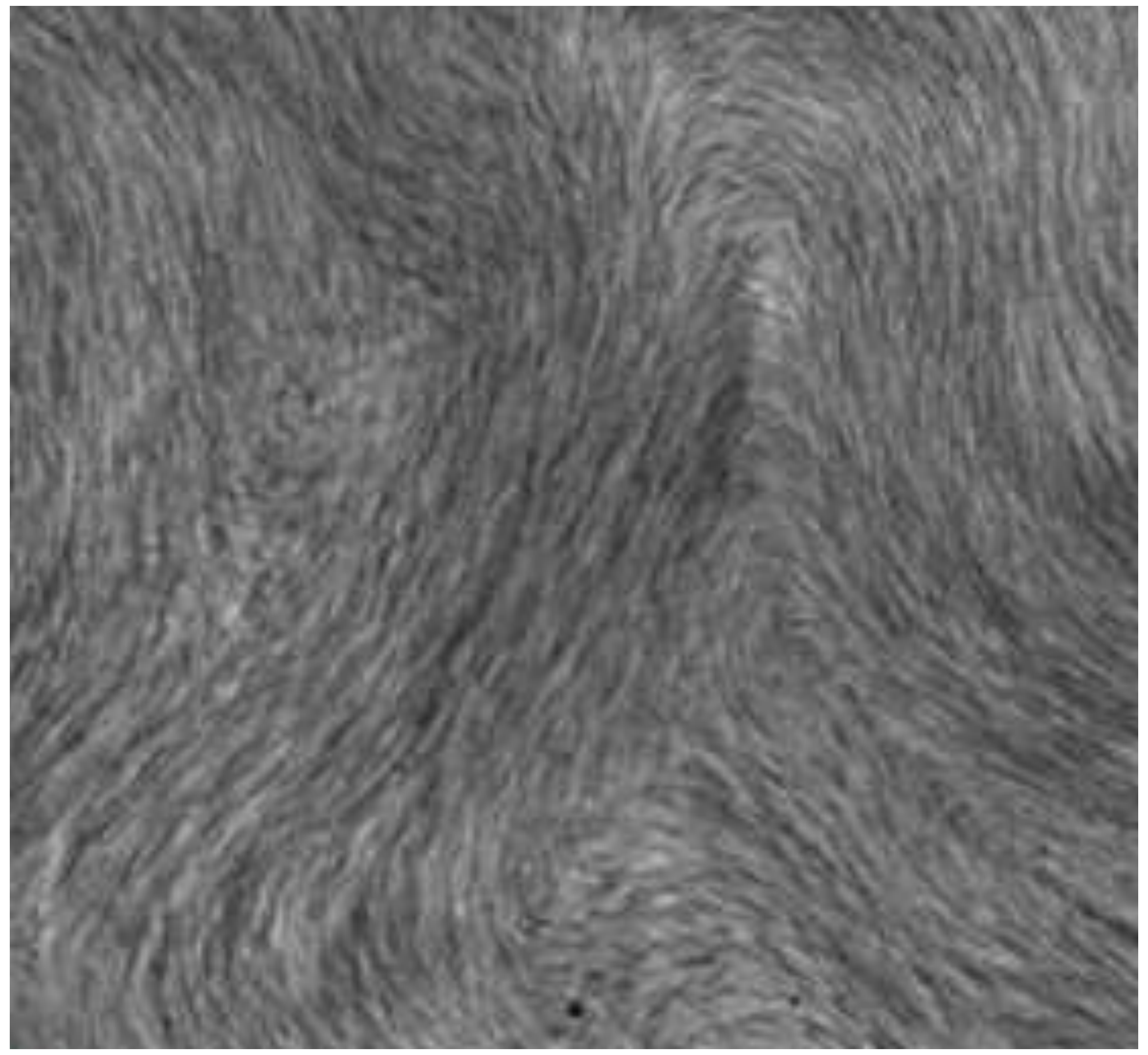
etc.

$$\left.\begin{array}{l}\text{Symmetric under } \dfrac{2\pi}{k} \text{ rotation} \\ \text{+ } \cancel{\text{reflection symmetries}}\end{array}\right\} \text{Dihedral group } D_n \quad \text{Cyclic group } C_n$$



Two-dimensional dihedral ('k-atic') liquid crystals

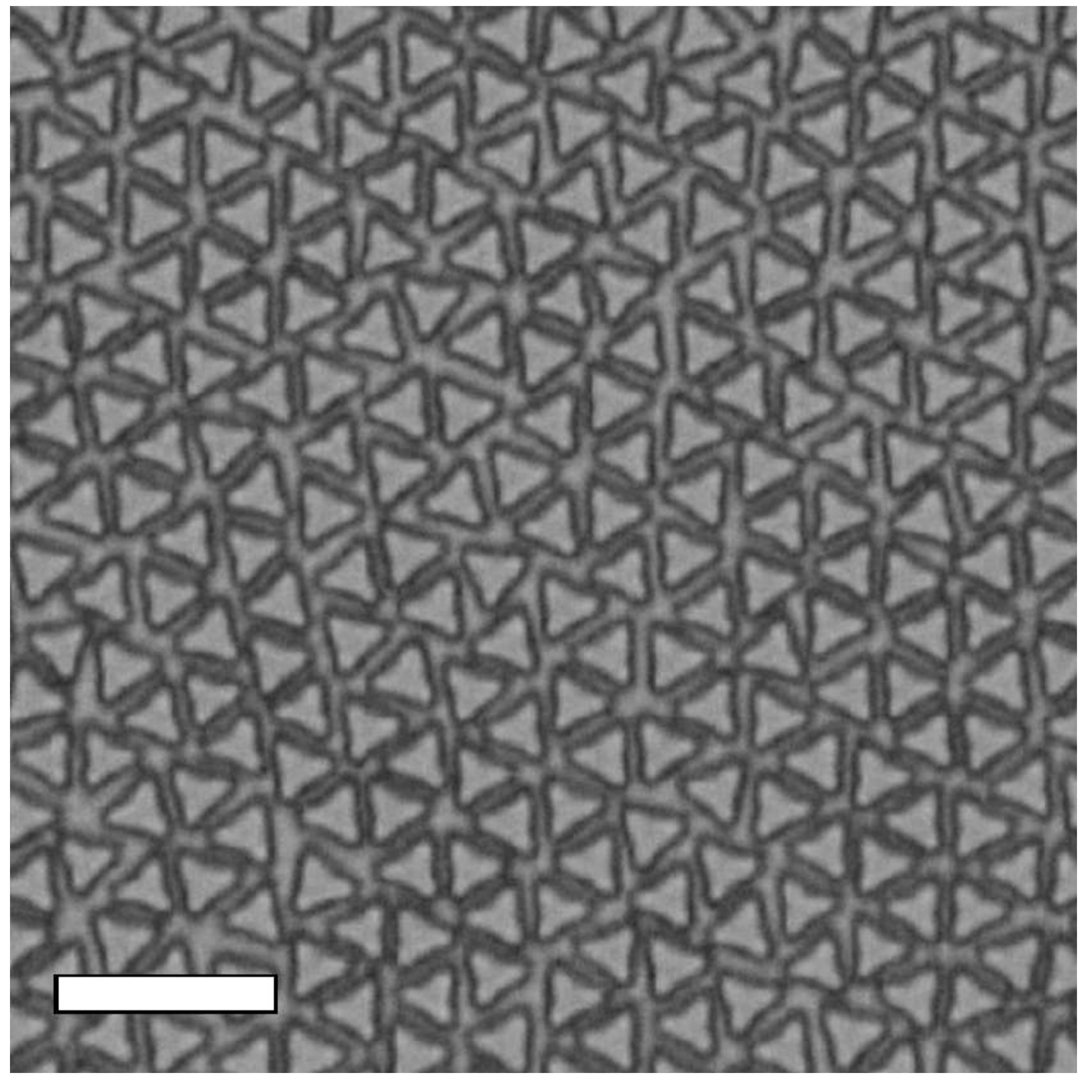
2-atic
(nematic)



Zhou et al. PNAS 2014

bacteria

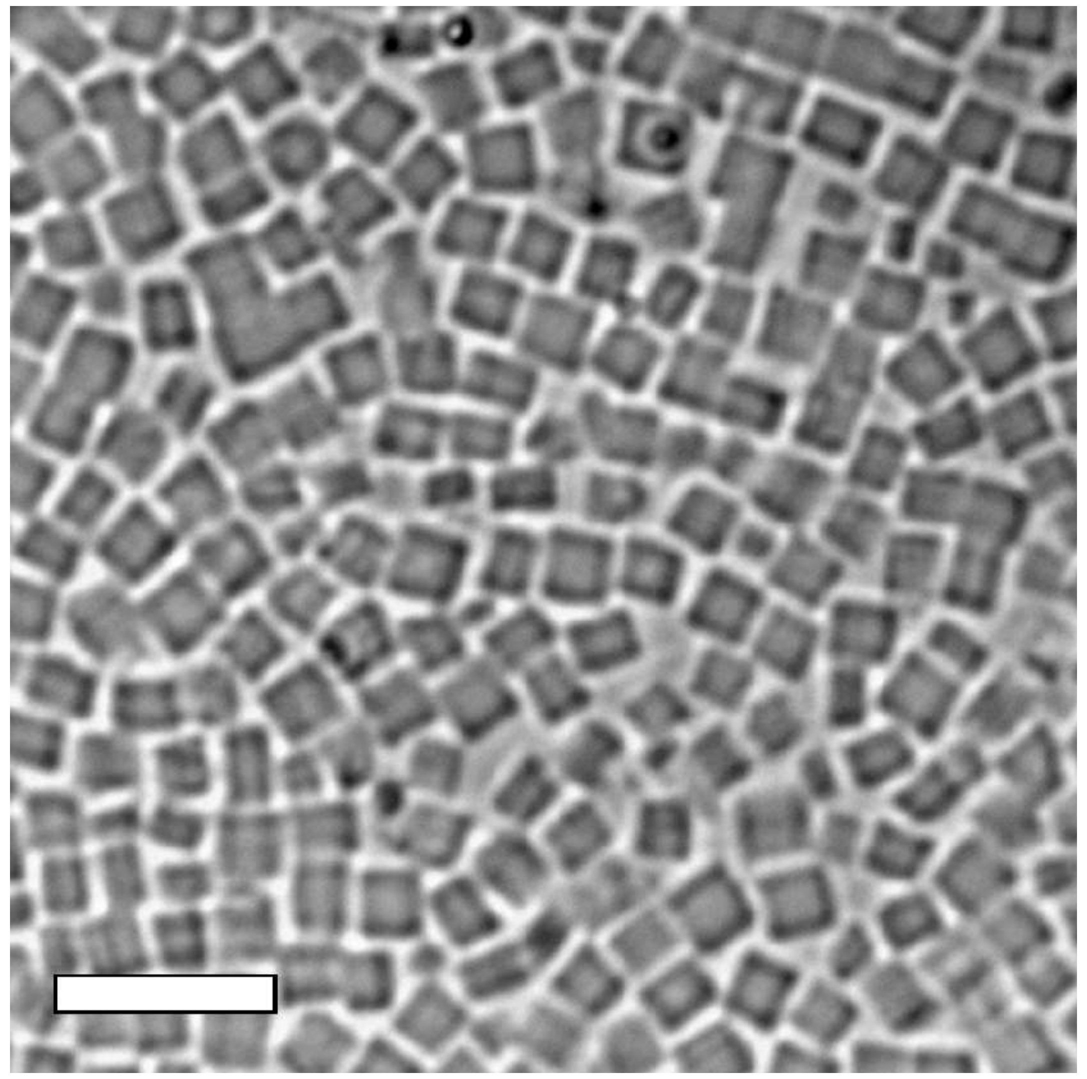
3-atic



Zhao et al. Nat Comm 2012

colloids

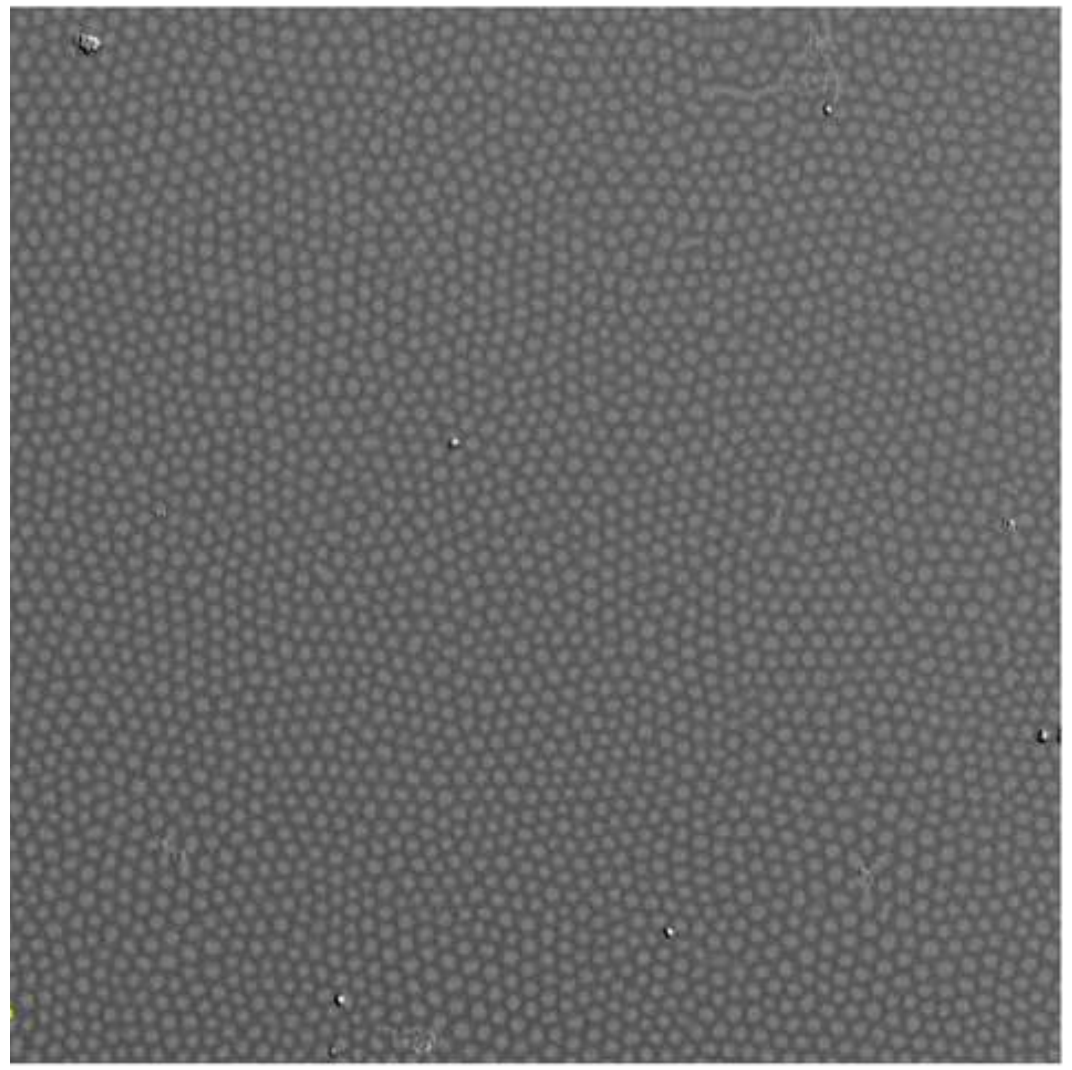
4-atic



Löffler. Thesis 2018 (Konstanz)

colloids

6-atic



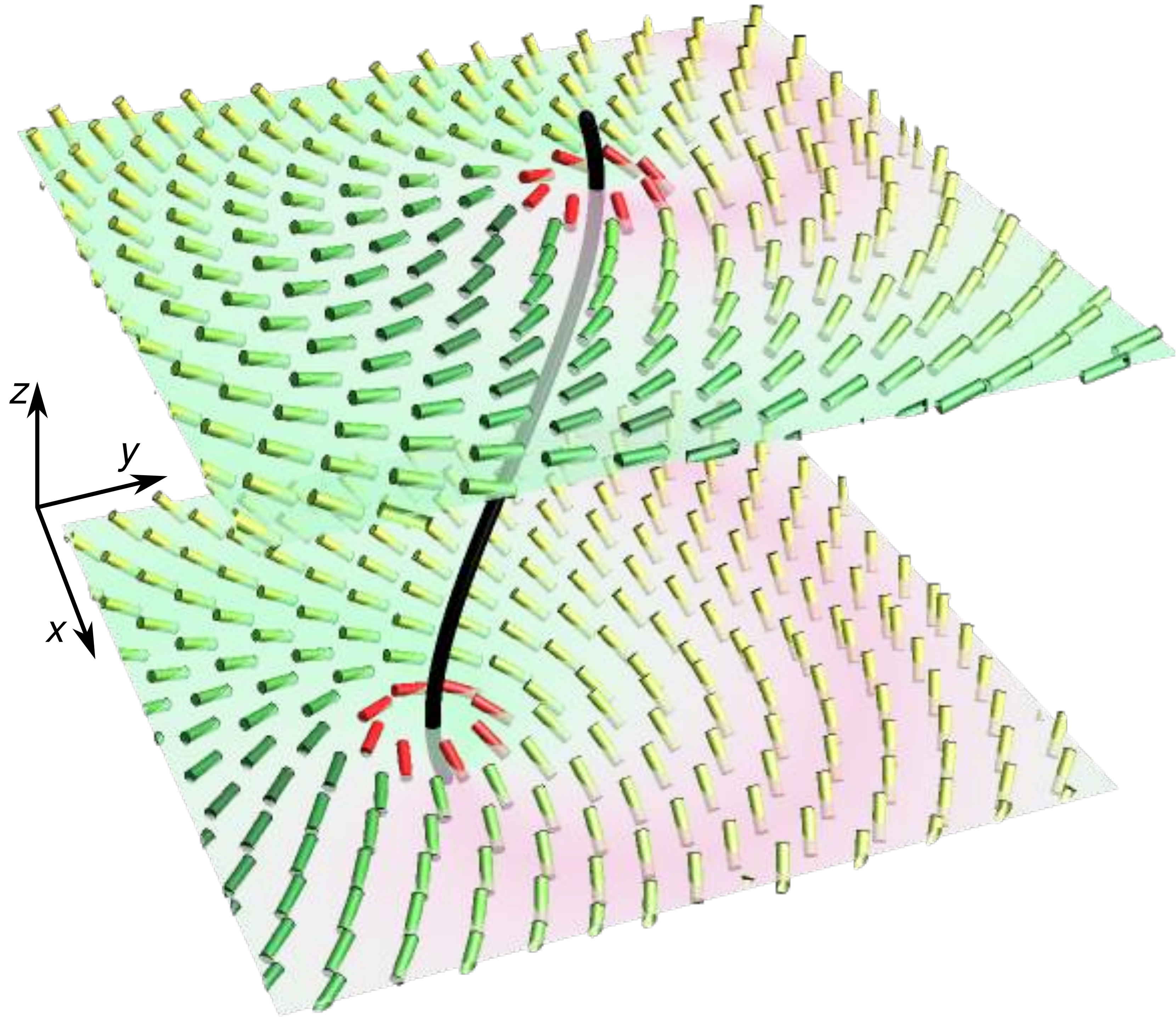
Zazvorka et al. Adv Funct Mater 2020

skyrmions

Positionally disordered but orientationally ordered

See also: Bowick & Giomi. Advances in Physics, 2009
Giomi, Toner & Sarkar. Phys Rev E 2022

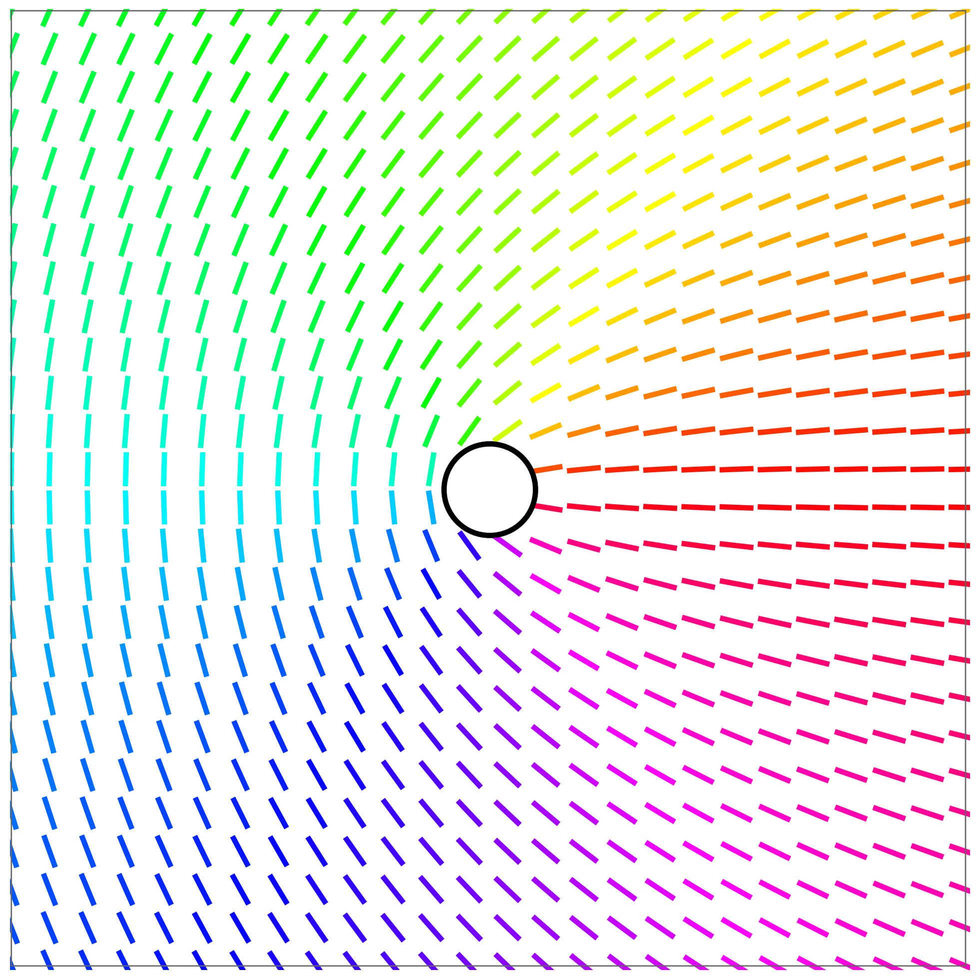
Topological defects in nematic liquid crystals



Uni-axial nematic LC

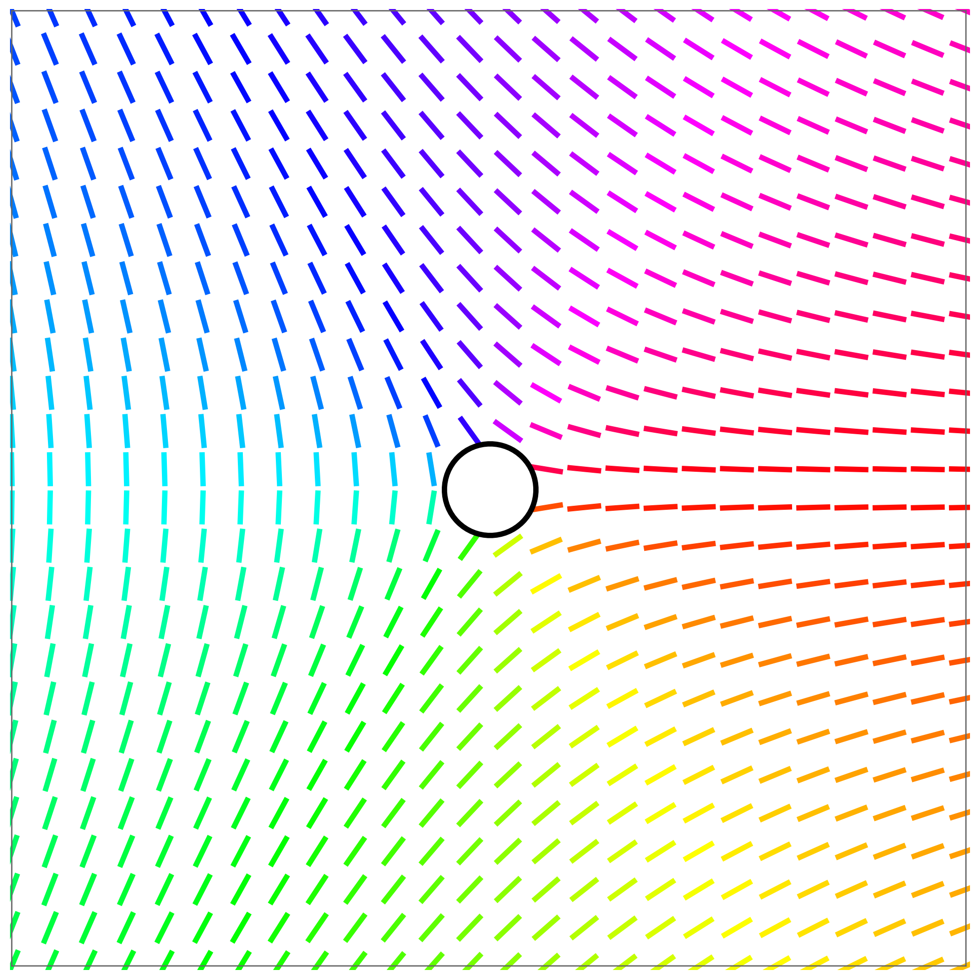
$$\mathbf{Q}(\boldsymbol{r}) = S(\boldsymbol{r})\left[\boldsymbol{n}(\boldsymbol{r}) \otimes \boldsymbol{n}(\boldsymbol{r}) - \frac{\mathbf{I}}{d}\right]$$

$q = +1/2$



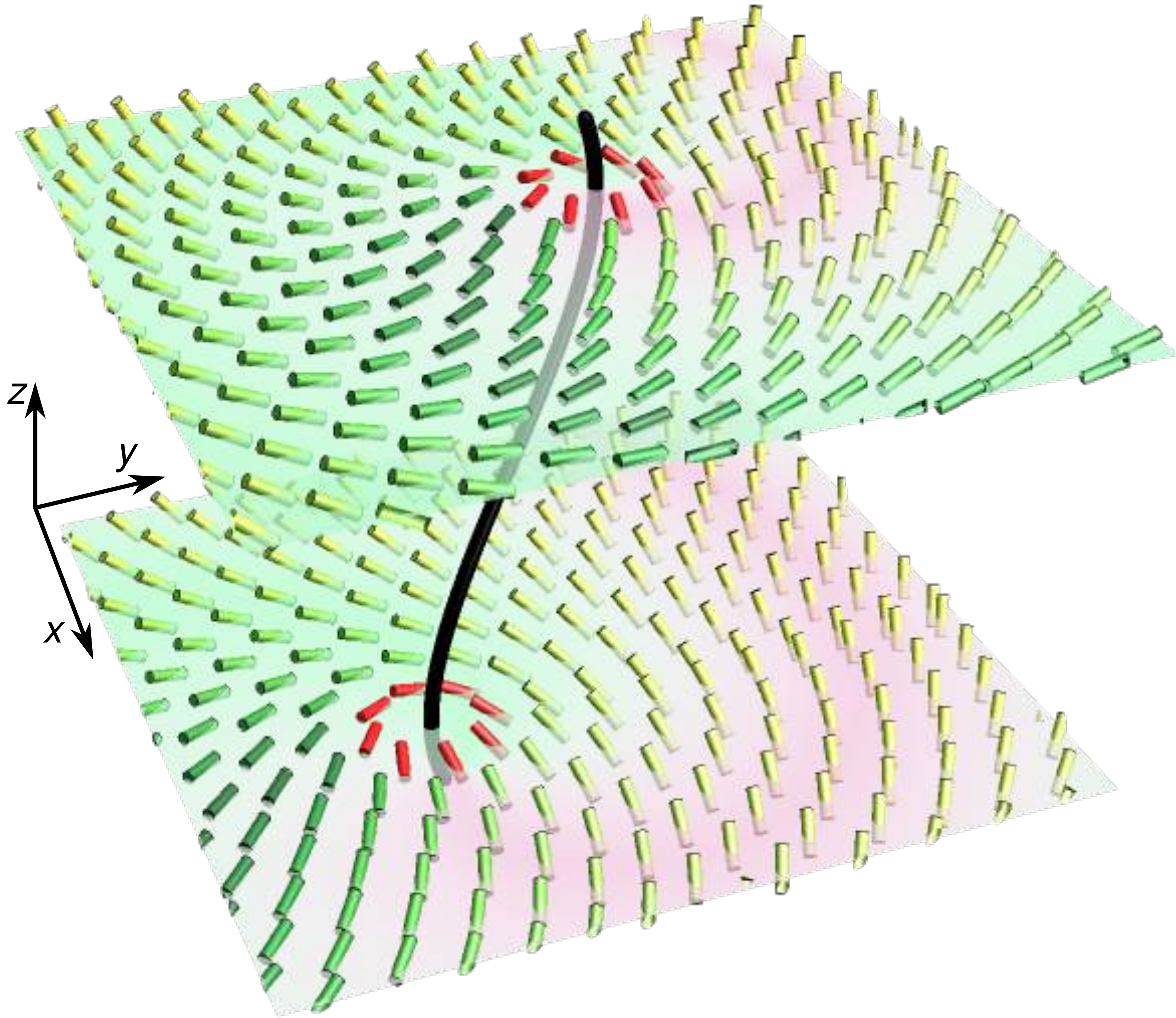
$$\boldsymbol{n}_+(\boldsymbol{r}) = (\cos\frac{\phi}{2}, \sin\frac{\phi}{2}, 0)$$

$q = -1/2$



$$\boldsymbol{n}_-(\boldsymbol{r}) = (\cos\frac{\phi}{2}, \sin\frac{-\phi}{2}, 0)$$

Topological defects in nematic liquid crystals



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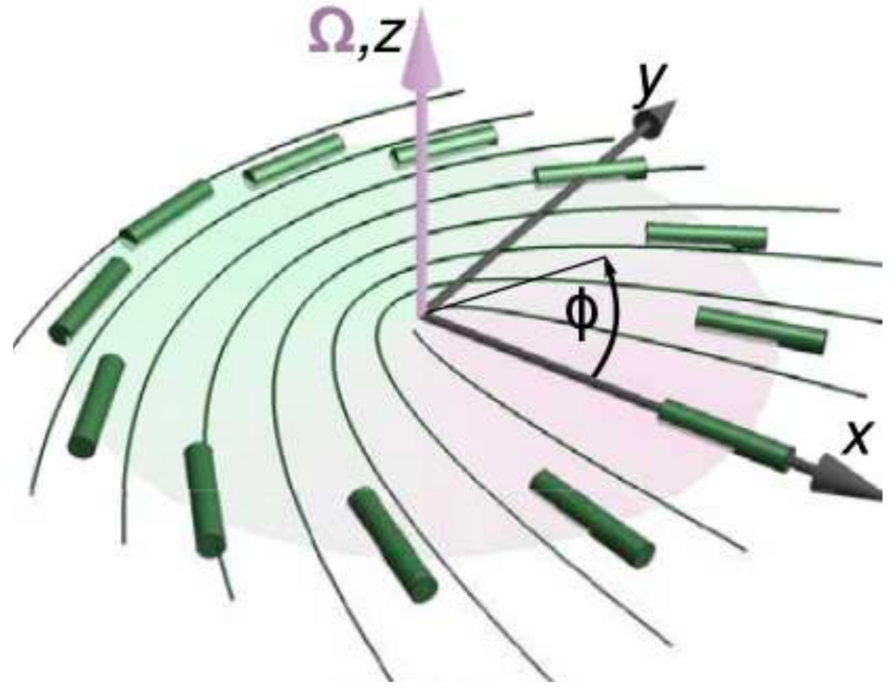

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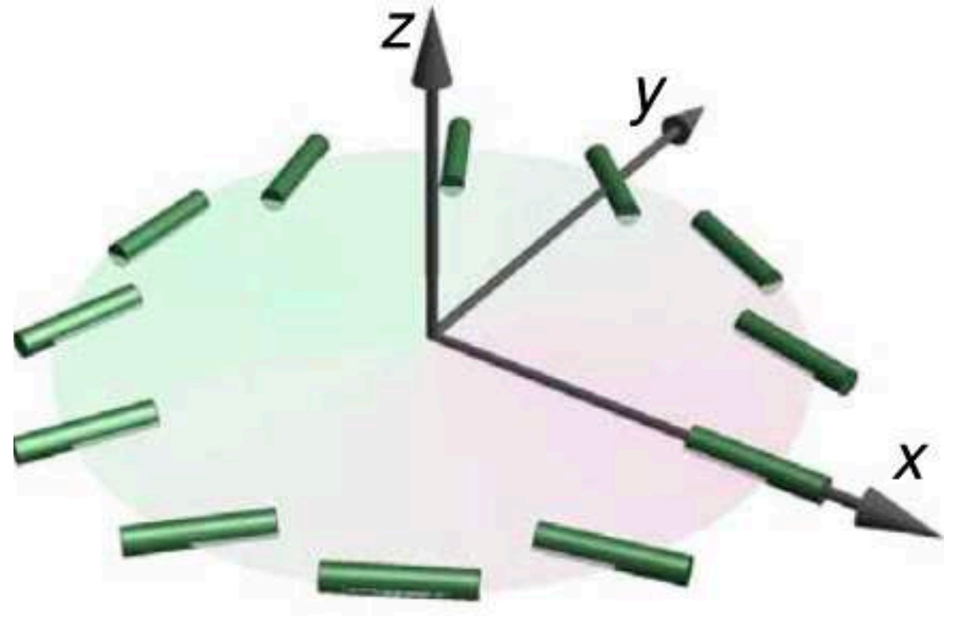


$$\boldsymbol{n}_-(\boldsymbol{r}) = (\cos\frac{\phi}{2}, \sin\frac{-\phi}{2}, 0)$$

Nematic bits (nbits)

Ziga Kos

Write director in Pauli-matrix basis

$$\boldsymbol{n}_0(r) = \left(\cos\frac{\phi}{2}\right)\boldsymbol{\sigma}_x + \left(\sin\frac{\phi}{2}\right)\boldsymbol{\sigma}_y + 0\,\boldsymbol{\sigma}_z$$

$$\Downarrow$$

Other director fields obtained by quaternion transformation

$$\boldsymbol{n}(\boldsymbol{r}) = \eta \boldsymbol{n}_0(\boldsymbol{r})\eta^\dagger$$

Unit-norm quaternion

$$\eta = \left(\cos\frac{\theta}{2}\right)\mathbf{1} + i\left(\sin\frac{\theta}{2}\right)(\hat{a}_x\boldsymbol{\sigma}_x + \hat{a}_y\boldsymbol{\sigma}_y + \hat{a}_z\boldsymbol{\sigma}_z)$$

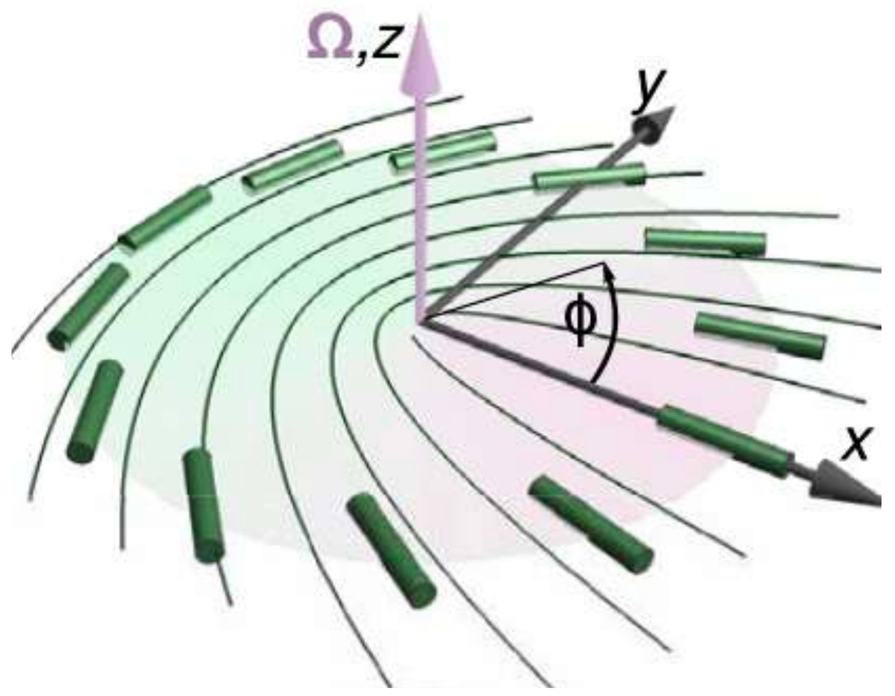
$$\Updownarrow$$

Unit-norm quaternions equivalent to SU(2)

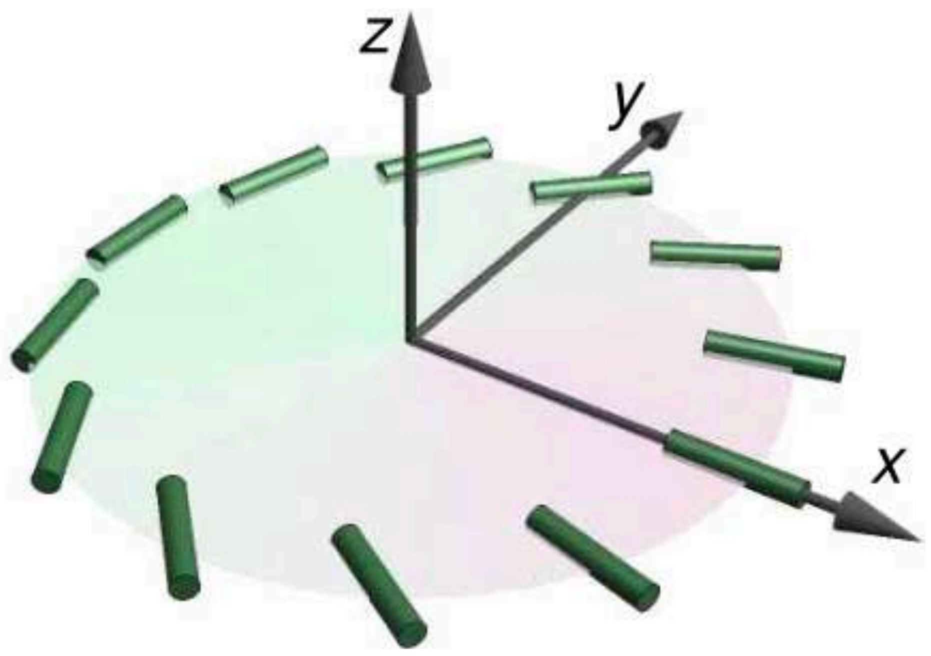
$$\begin{pmatrix} c_1 & -c_2^* \\ c_2 & c_1^* \end{pmatrix} \qquad |c_1|^2 + |c_2|^2 = 1$$

Fully specified by first column

$$|\eta) = c_1\,|0) + c_2\,|1) = e^{i\frac{\gamma}{2}}\left[\cos\frac{\beta}{2}\,|0) + e^{-i\alpha}\sin\frac{\beta}{2}\,|1)\right]$$



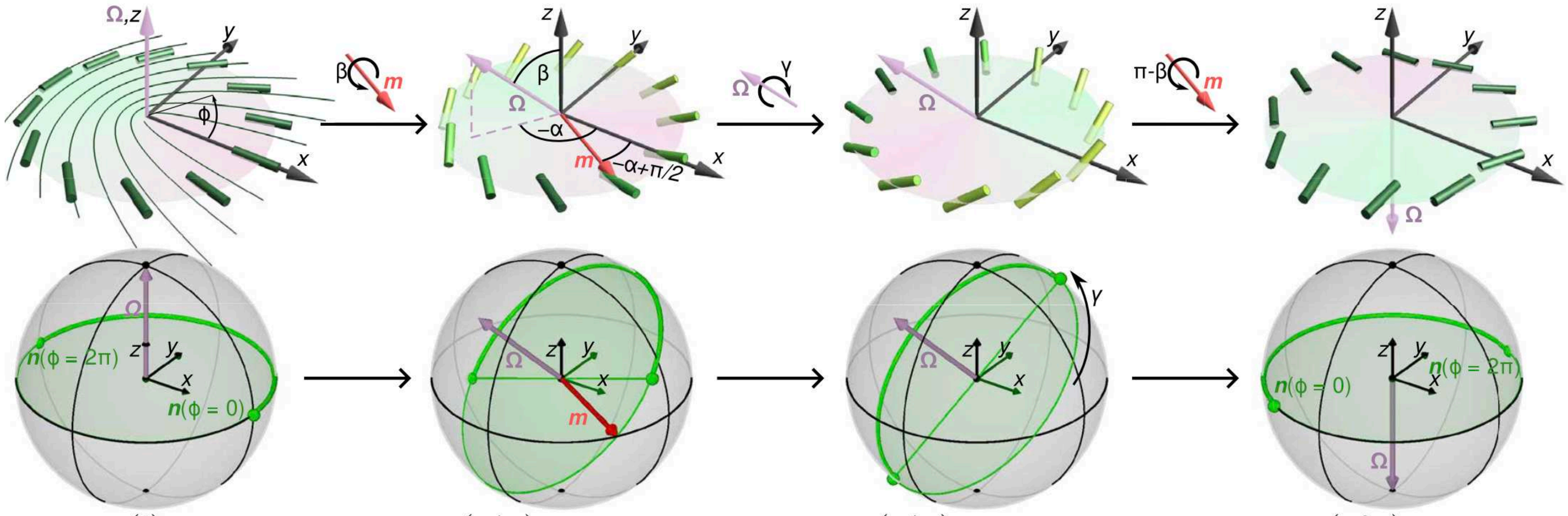
$$|0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$



$$|1) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Kos & JD, Science Advances 2022

$$|\eta) = c_1\,|0) + c_2\,|1) = e^{i\frac{\gamma}{2}}\left[\cos\frac{\beta}{2}\,|0) + e^{-i\alpha}\sin\frac{\beta}{2}\,|1)\right]$$

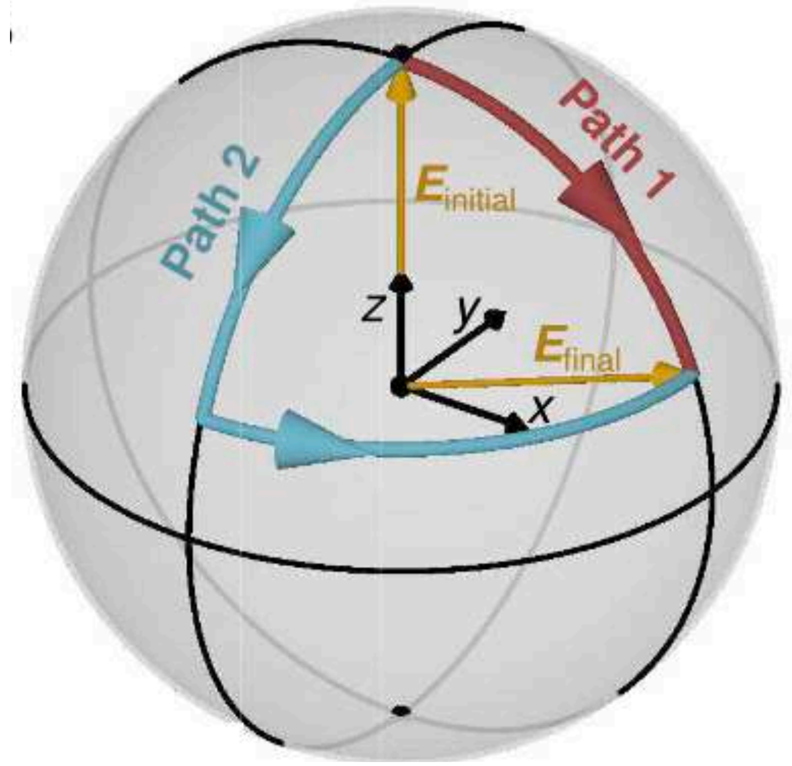


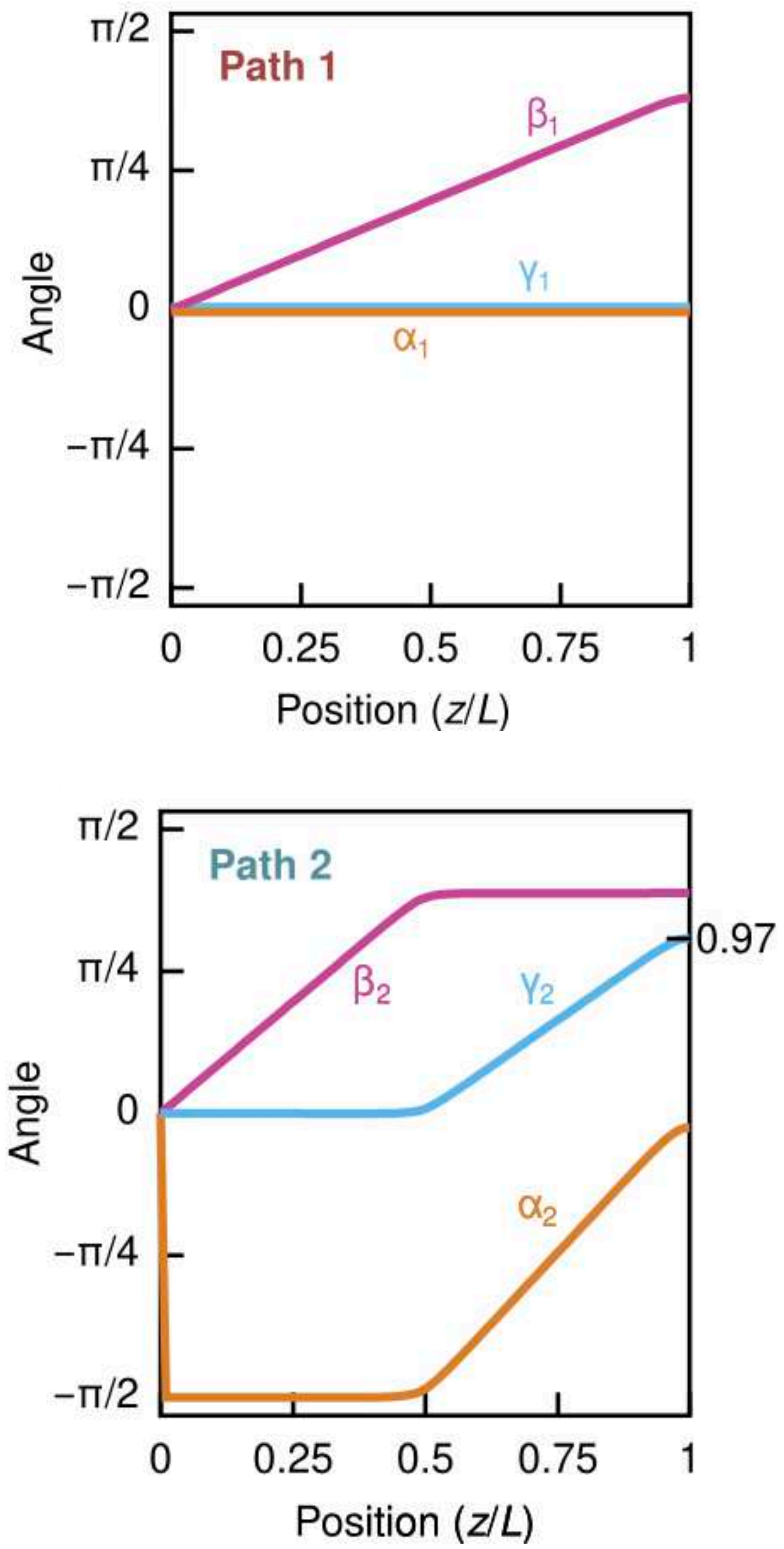
$|\eta) = \begin{pmatrix}1\\0\end{pmatrix} = |0)$ $\quad |\eta) = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\e^{2\pi i/3}\end{pmatrix} = \frac{1}{\sqrt{2}}\left(|0) + e^{2\pi i/3}|1)\right)$ $\quad |\eta) = \frac{e^{i\pi/6}}{\sqrt{2}}\begin{pmatrix}1\\e^{2\pi i/3}\end{pmatrix} = \frac{e^{i\pi/6}}{\sqrt{2}}\left(|0) + e^{2\pi i/3}|1)\right)$ $\quad |\eta) = \begin{pmatrix}0\\e^{5\pi i/6}\end{pmatrix} = e^{5\pi i/6}|1)$

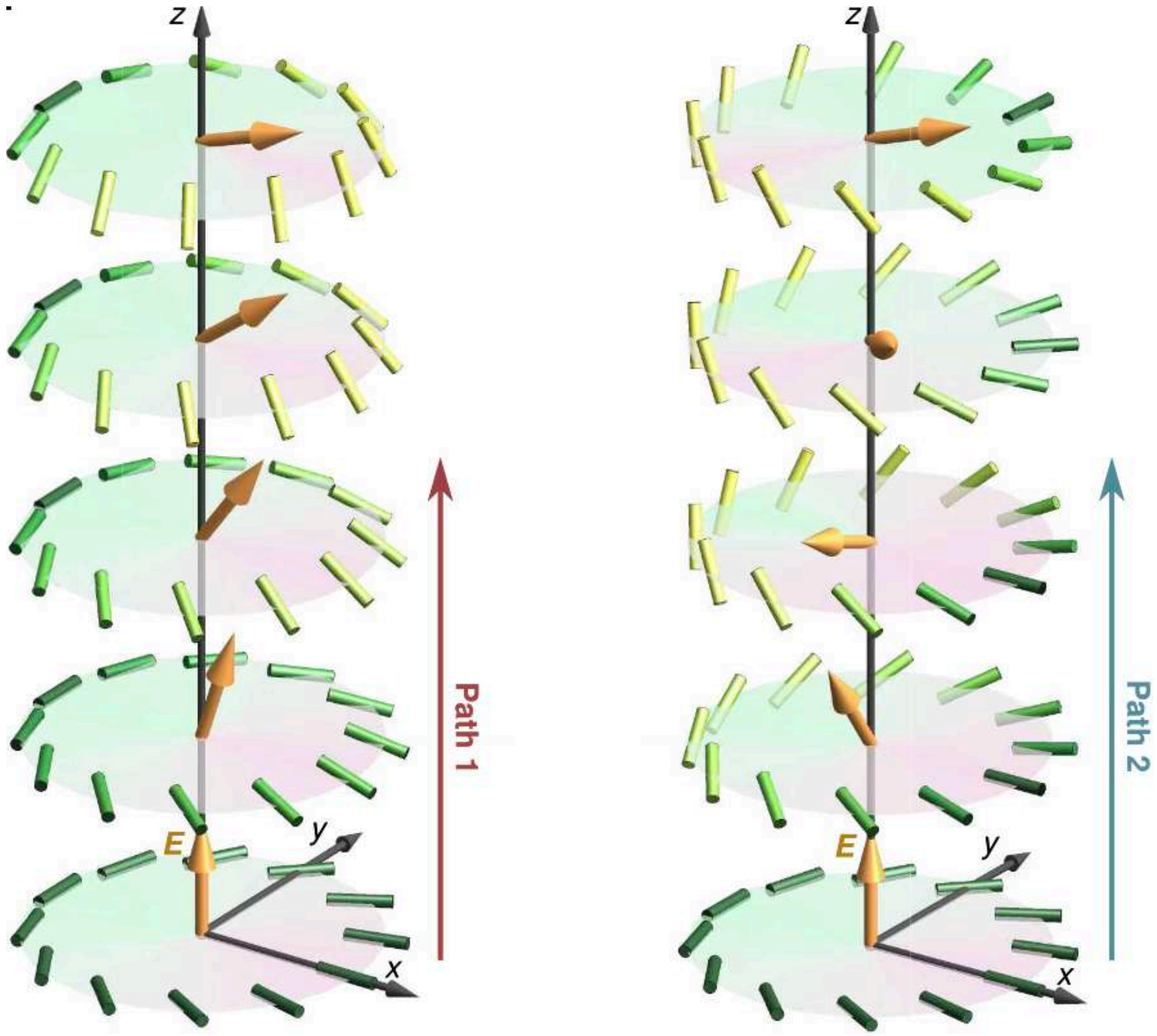
Kos & JD, Science Advances 2022

nbit manipulations with electric fields

$$\frac{\mathrm{d}\eta}{\mathrm{d}t} = \frac{K}{\Gamma}\frac{\partial^2 \eta}{\partial z^2} - \frac{\epsilon_{\mathrm{a}}}{4\Gamma}\,\mathrm{Tr}\left(\boldsymbol{E}\eta\sigma_z\eta^\dagger\right)\boldsymbol{E}\eta\sigma_z - \tilde{\lambda}\eta, \qquad \tilde{\lambda} = \frac{\lambda}{\pi\Gamma}$$

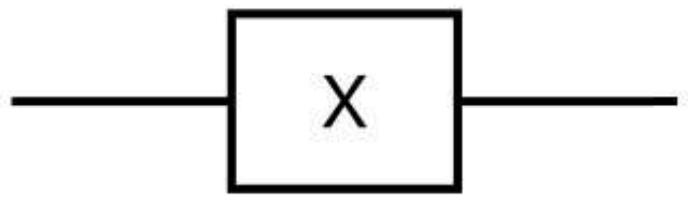




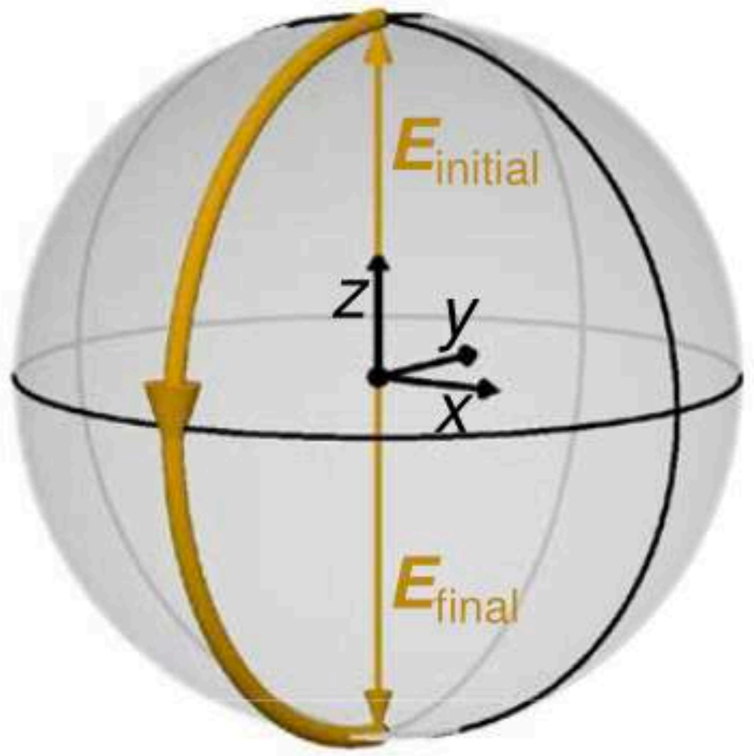


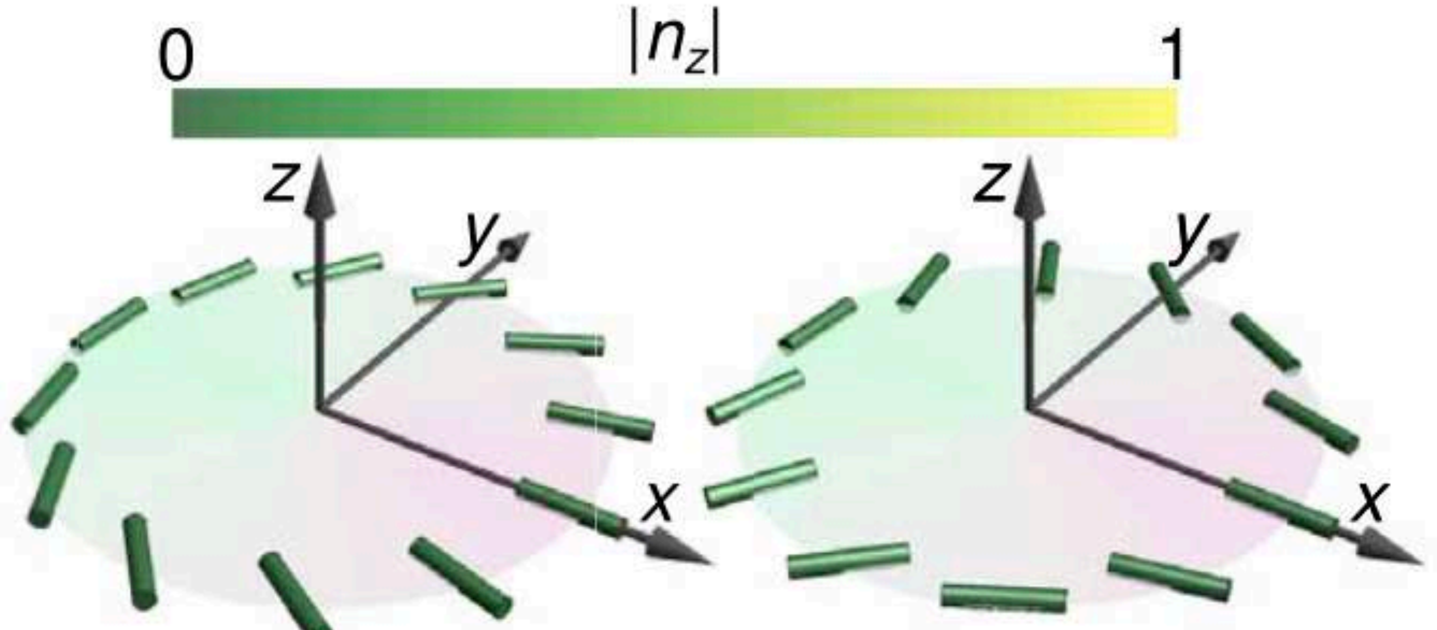
single nbit-operations

Pauli-X gate



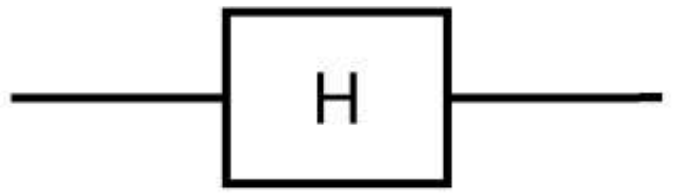
$$i\sigma_X = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$$



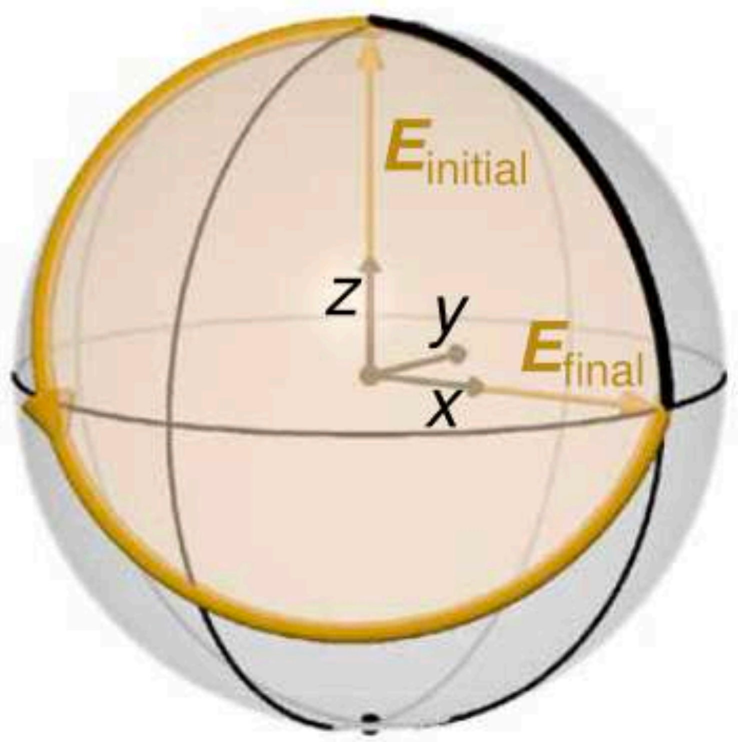


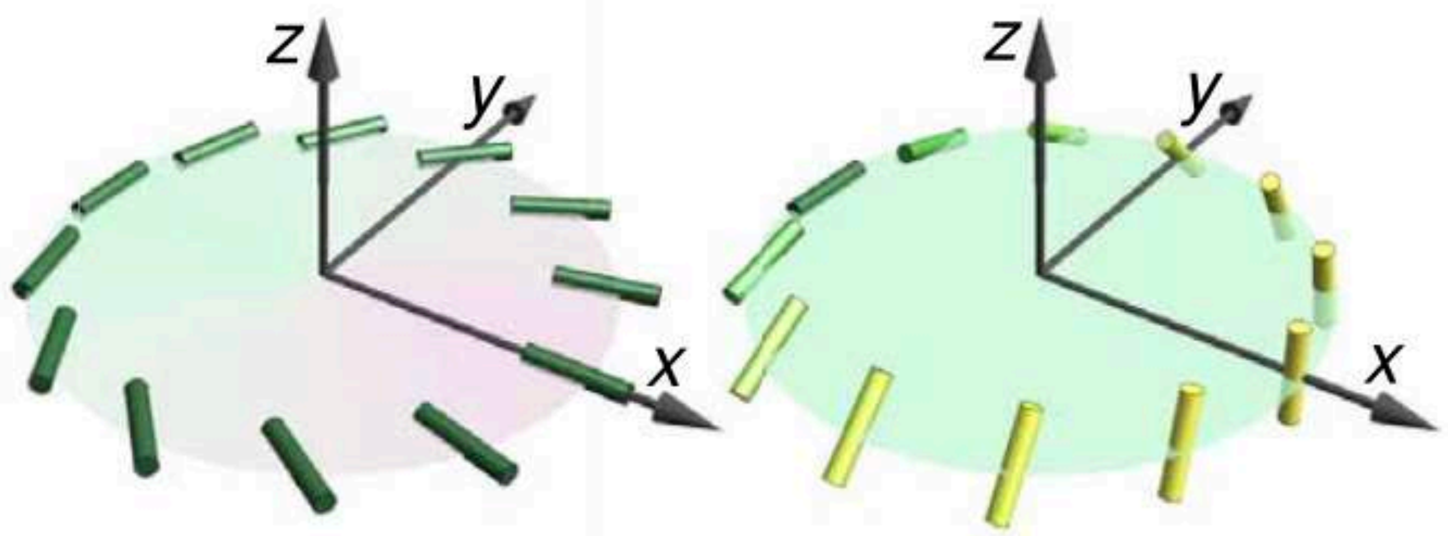
$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \xrightarrow{X} \begin{pmatrix} 0 \\ i \end{pmatrix}$$

Hadamard gate



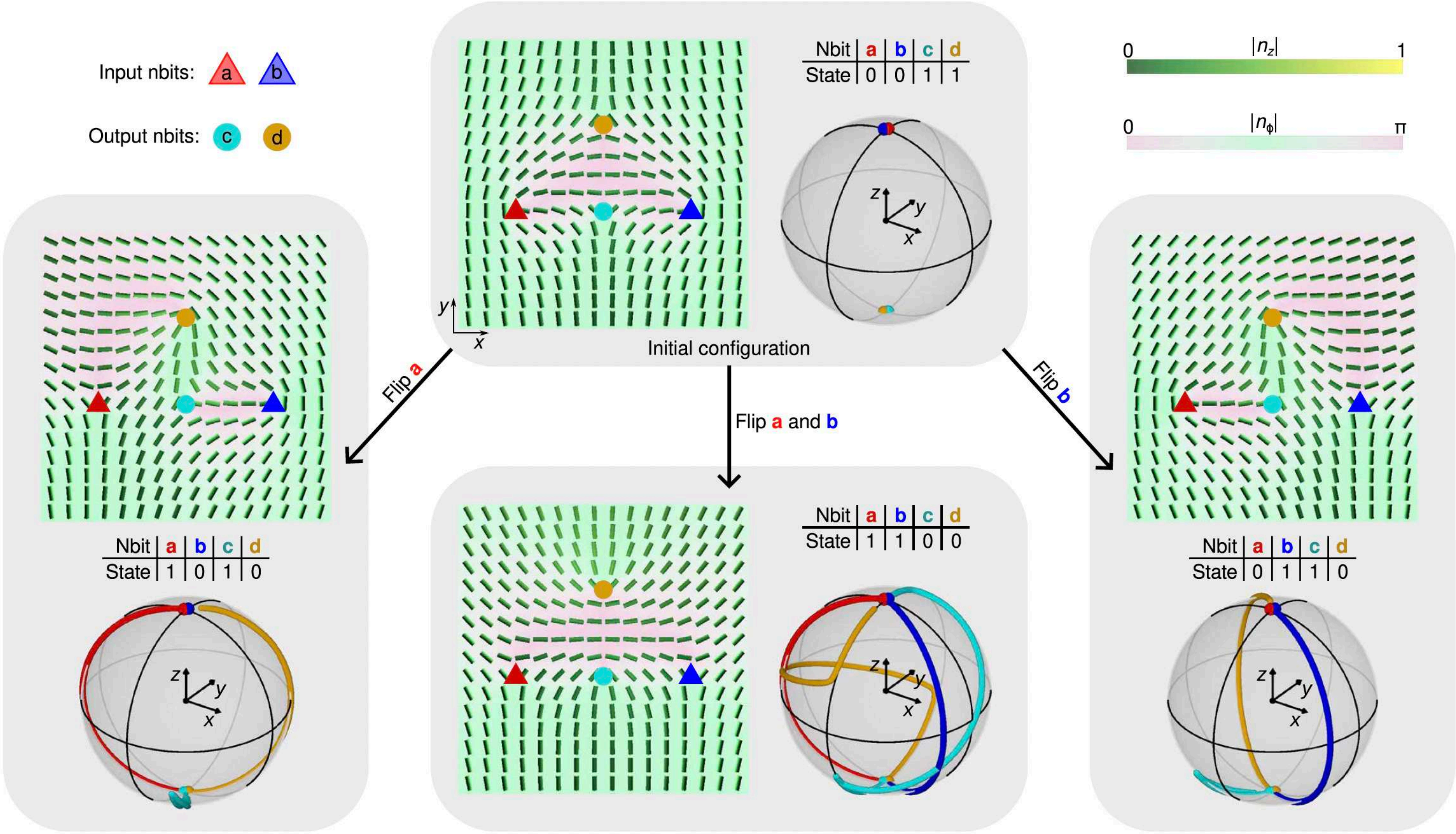
$$\frac{i}{\sqrt{2}}(\sigma_X + \sigma_Z) = \frac{1}{\sqrt{2}} \begin{pmatrix} i & i \\ i & -i \end{pmatrix}$$





$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \xrightarrow{H} \frac{1}{\sqrt{2}} \begin{pmatrix} i \\ i \end{pmatrix}$$

Classical nbit-logic : universal gates



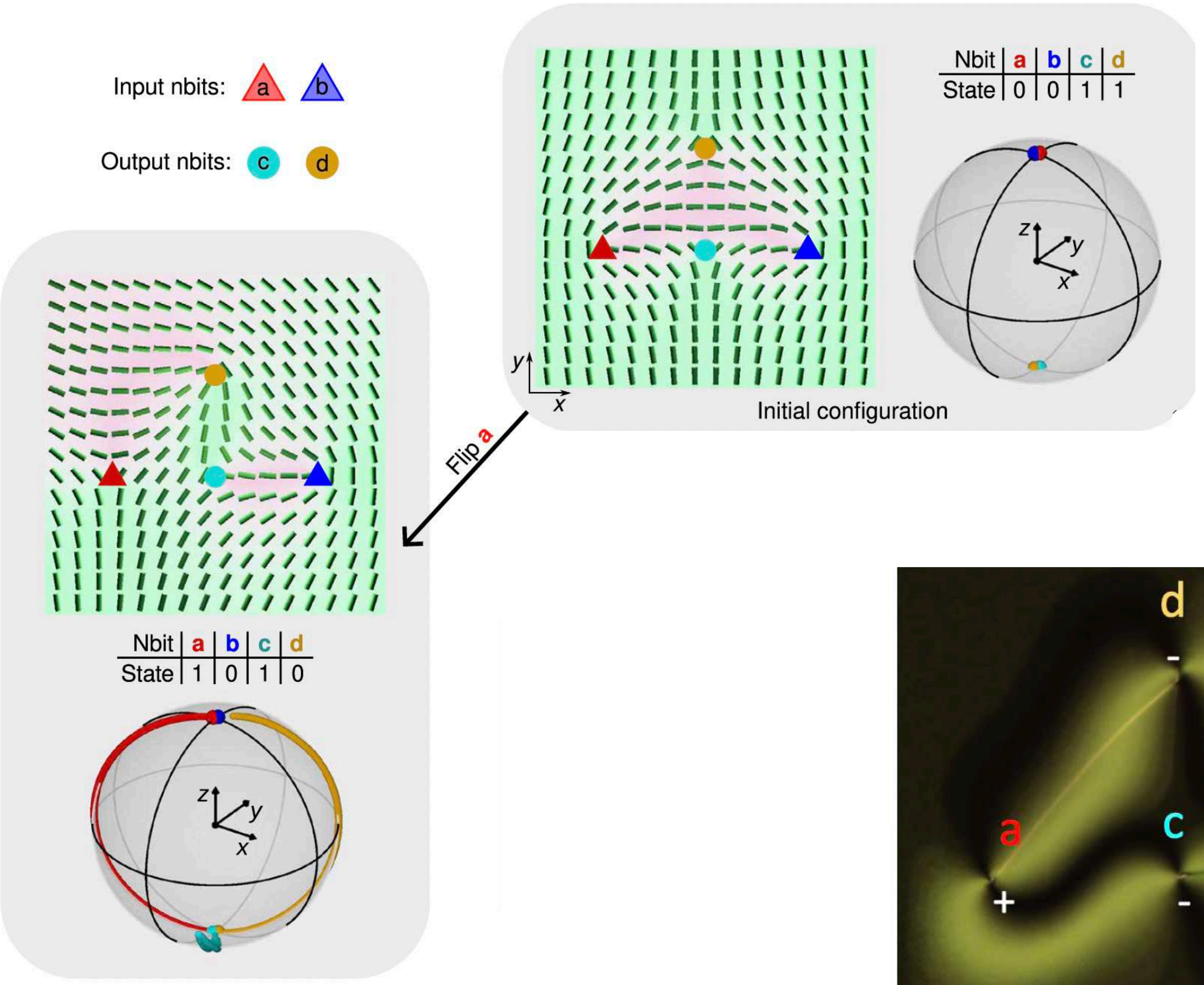
c: NAND

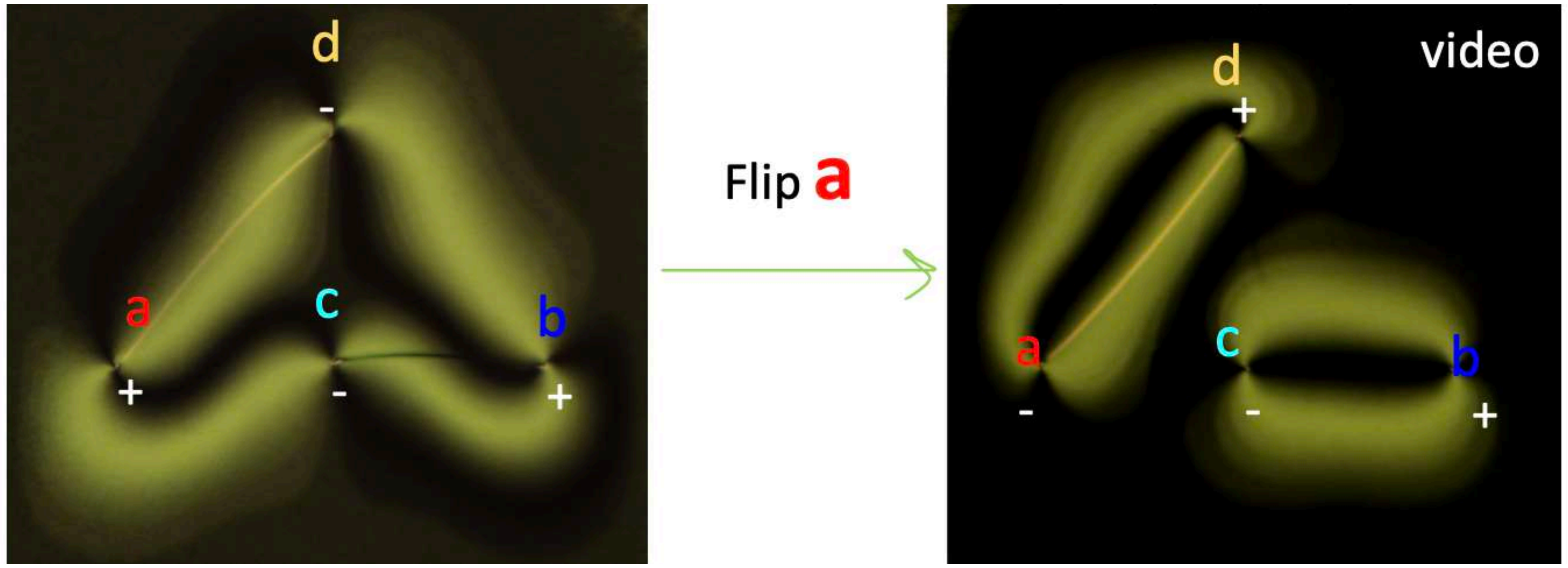
d: NOR

Kos & JD, Science Advances 2022

Recent experiments: towards nbit logic gates

Ivan Smalyukh
(CU Boulder)





Recent experiments: towards nbit memory

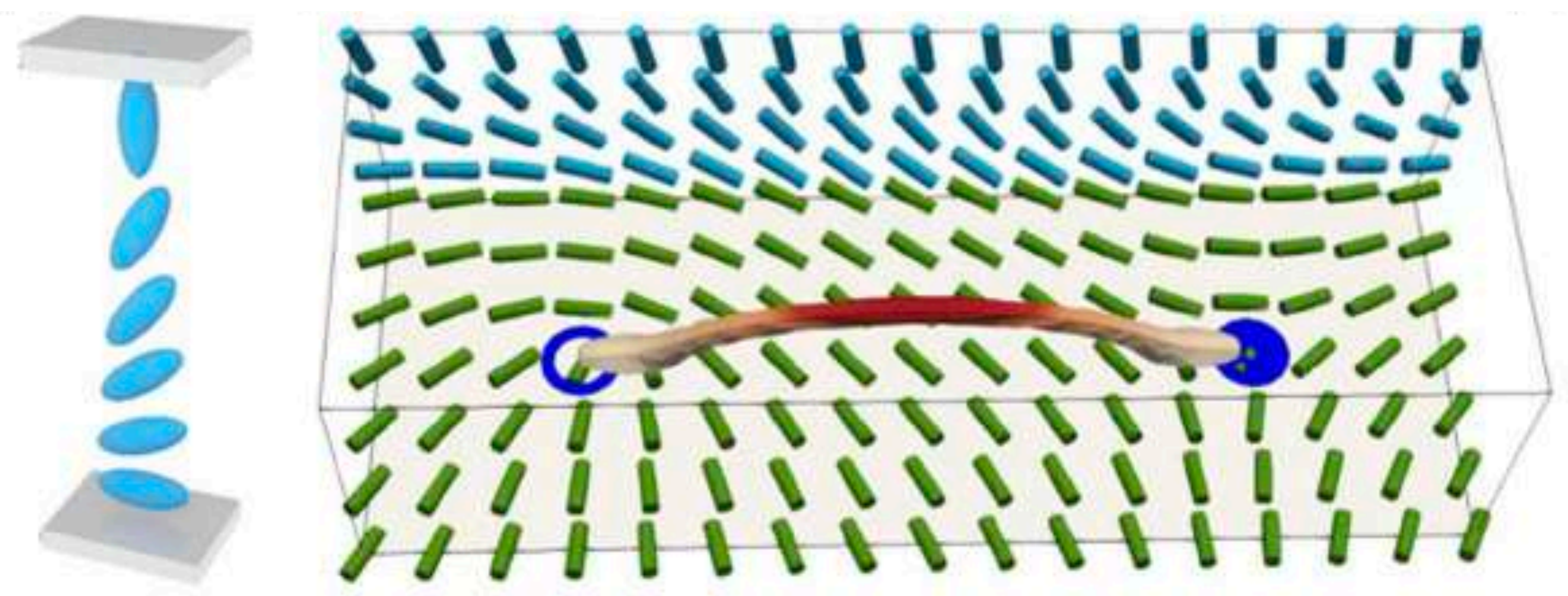
Ivan Smalyukh
(CU Boulder)

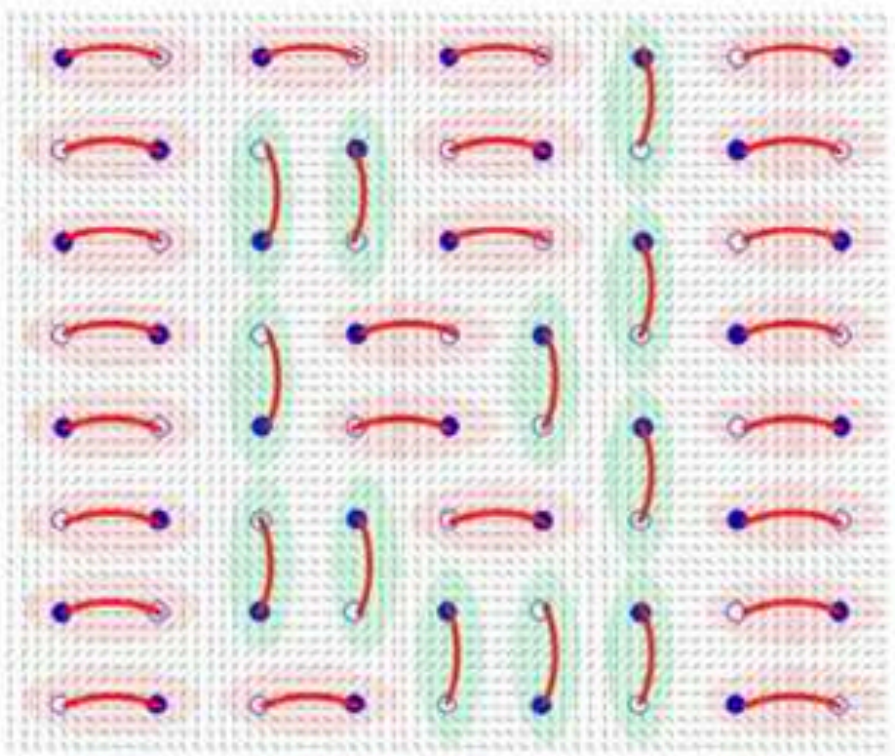
PHYSICAL REVIEW X **15**, 021084 (2025)

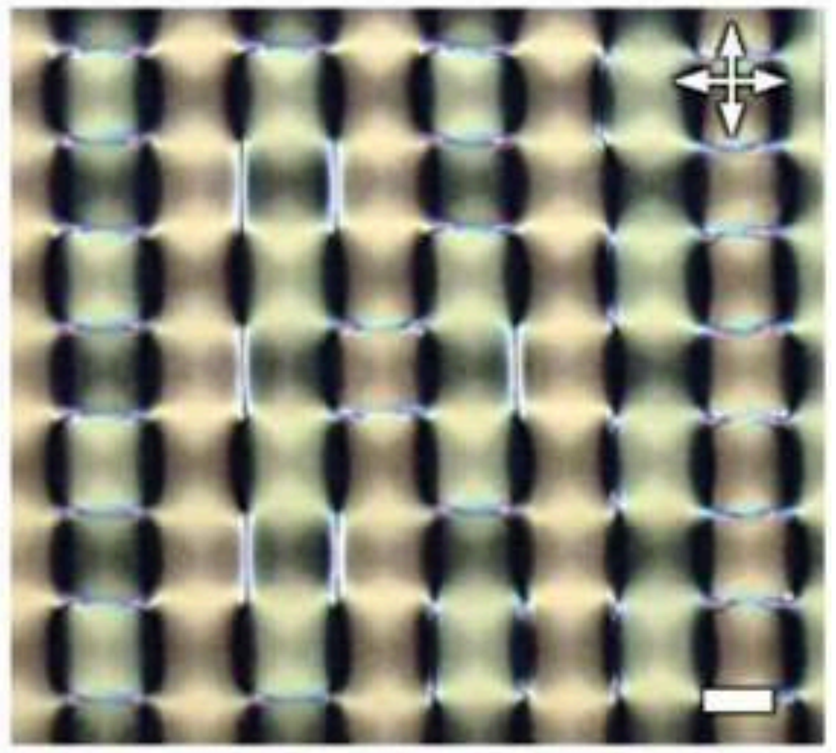
Featured in Physics

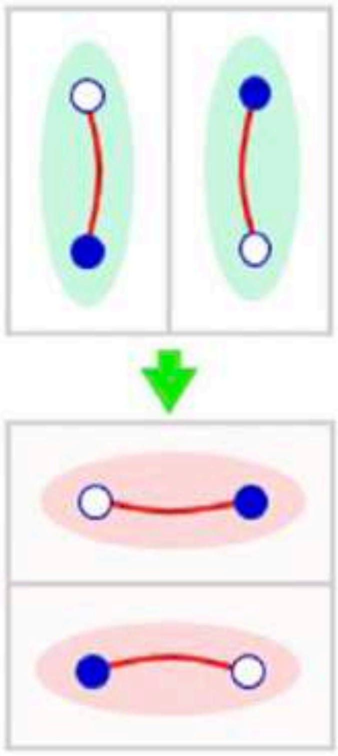
Emergent Dimer-Model Topological Order and Quasiparticle Excitations in Liquid Crystals: Combinatorial Vortex Lattices

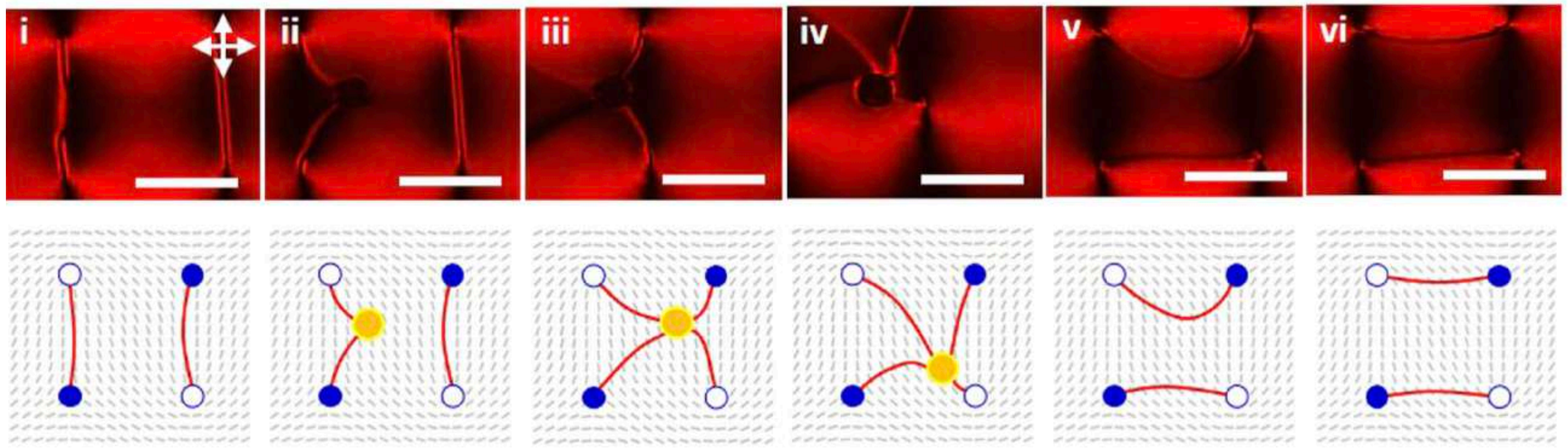
Cuiling Meng,[1] Jin-Sheng Wu,[1] Žiga Kos,[2,3,4,5] Jörn Dunkel,[2,3] Cristiano Nisoli,[6,*] and Ivan I. Smalyukh[1,2,7,8,†]







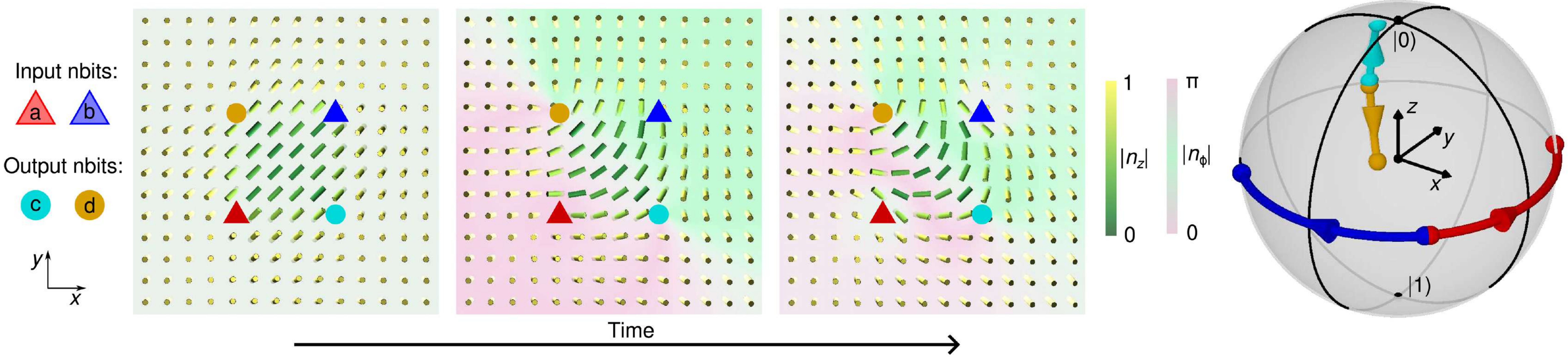




Outlook: beyond classical bit logic

Ziga Kos

$$|\eta) = c_1\,|0) + c_2\,|1) = e^{i\frac{\gamma}{2}}\left[\cos\frac{\beta}{2}\,|0) + e^{-i\alpha}\sin\frac{\beta}{2}\,|1)\right]$$

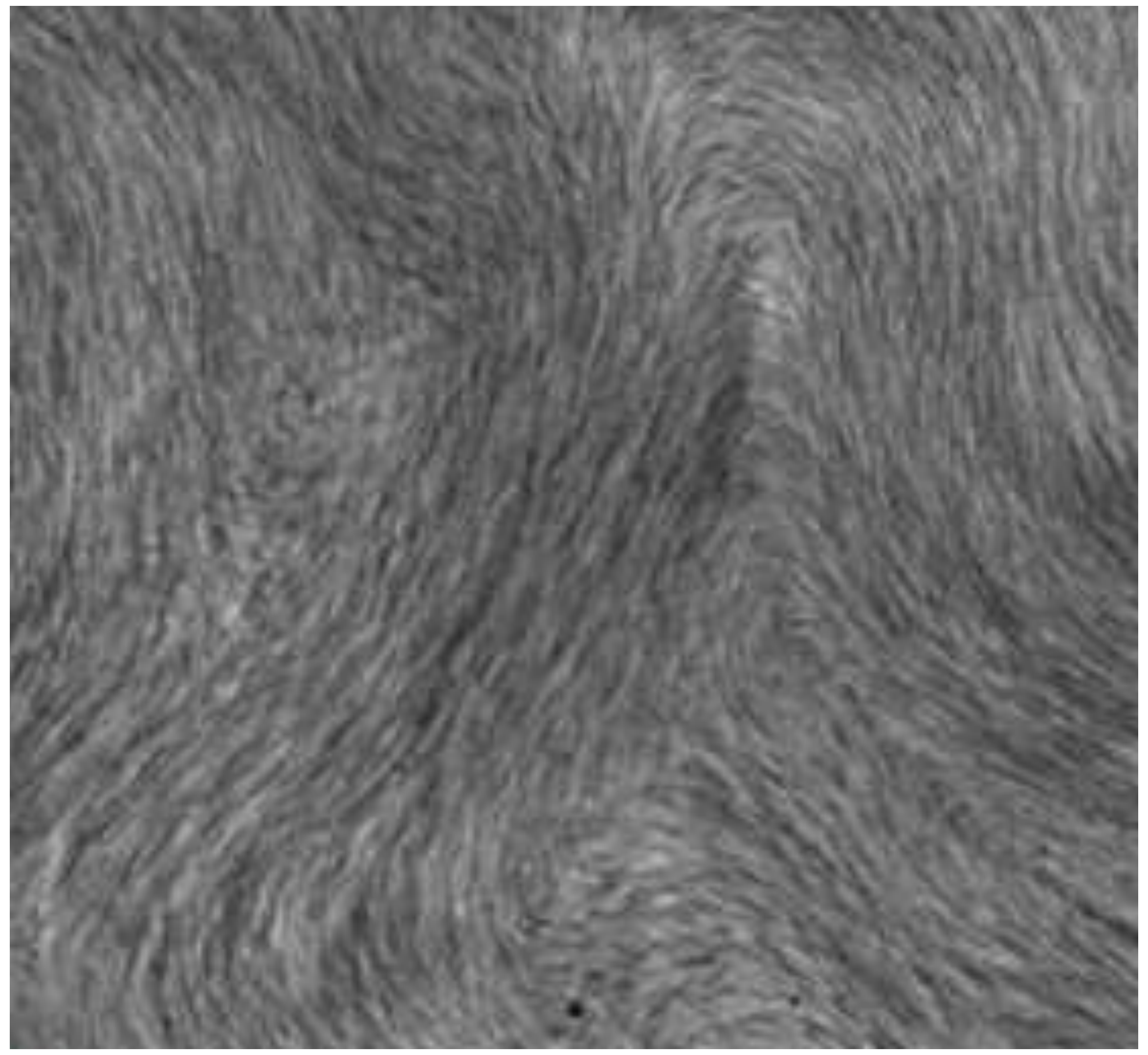


What is the space of controllable non-classical bit gate operations ?

Petur Bryde

Two-dimensional dihedral ('k-atic') liquid crystals

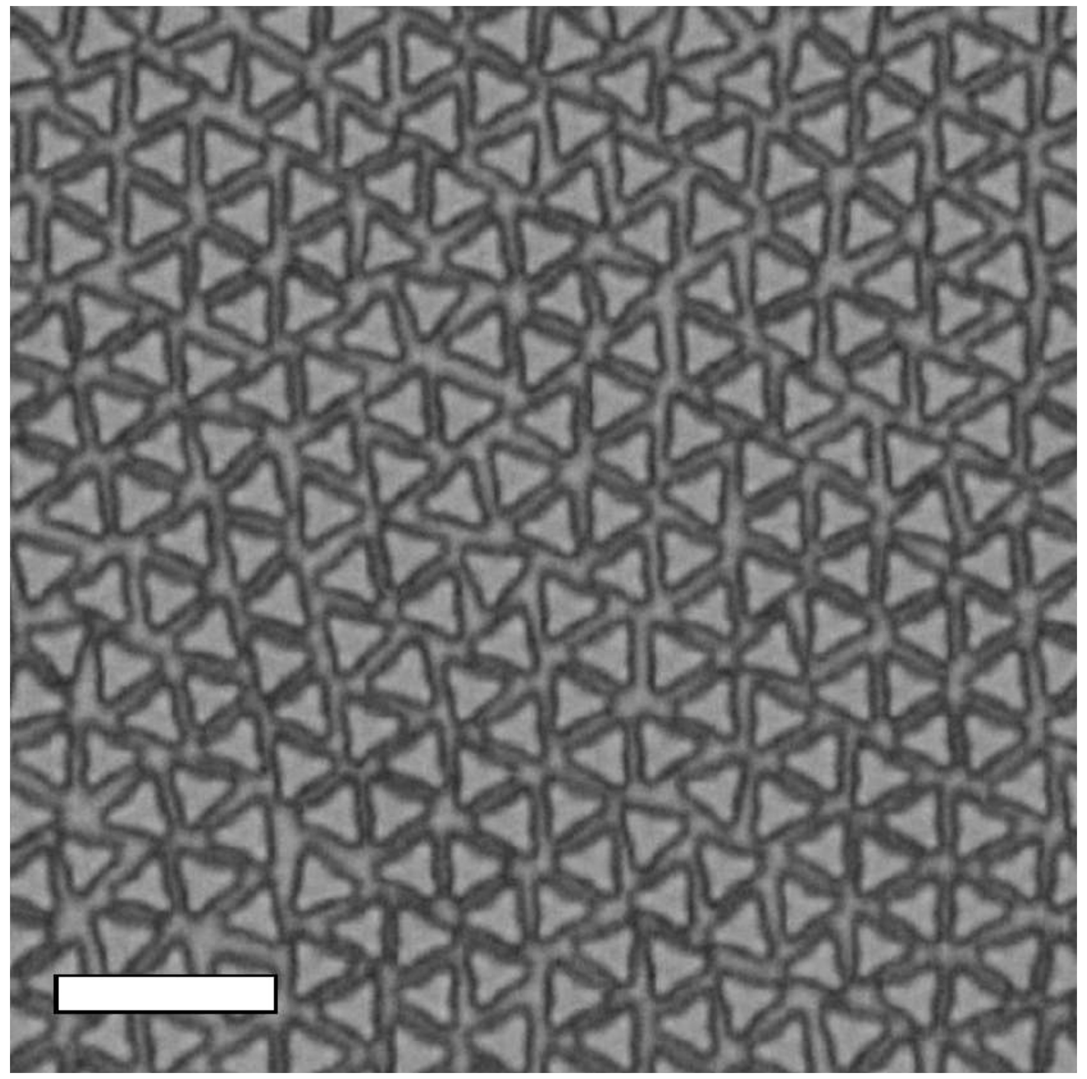
2-atic
(nematic)



Zhou et al. PNAS 2014

bacteria

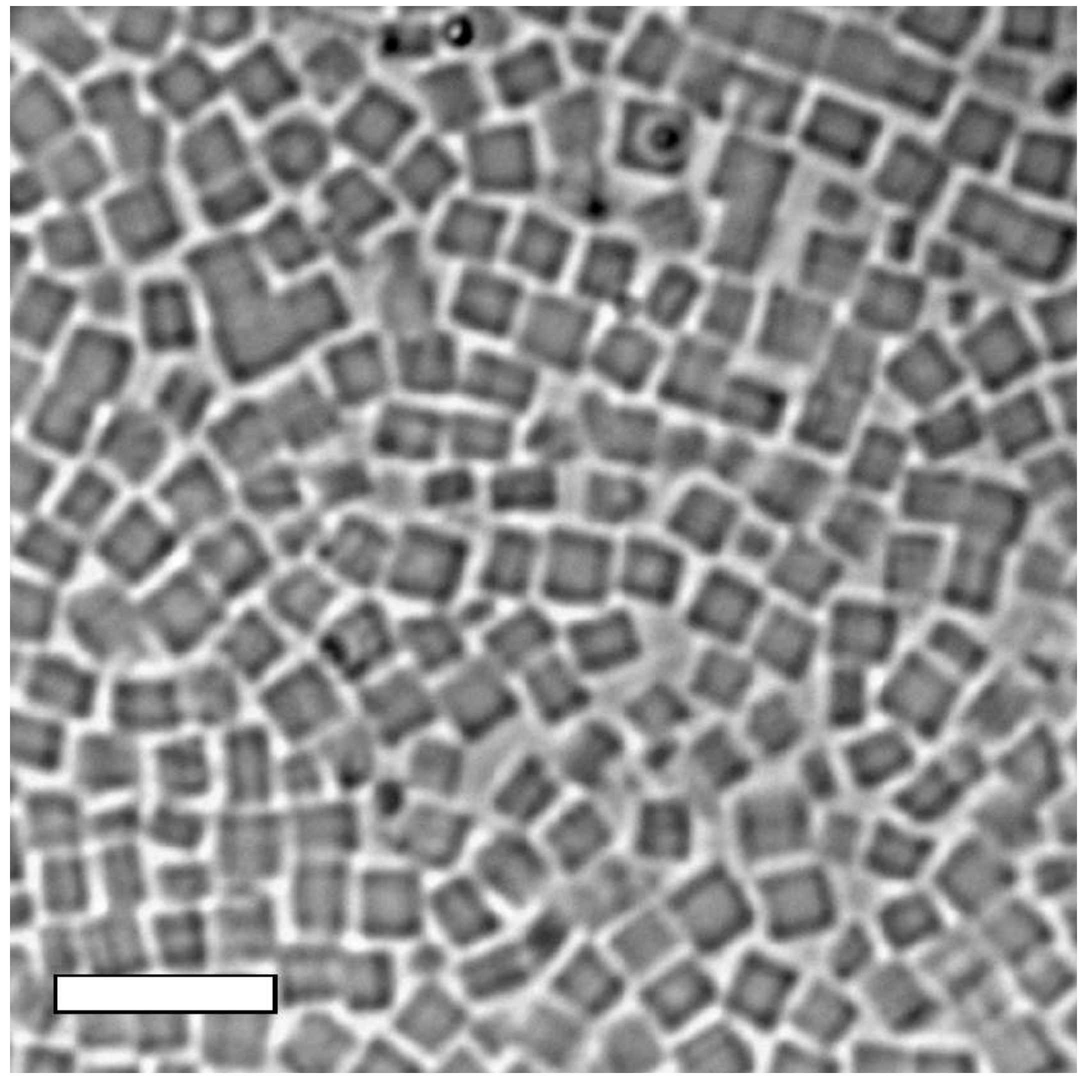
3-atic



Zhao et al. Nat Comm 2012

colloids

4-atic



Löffler. Thesis 2018 (Konstanz)

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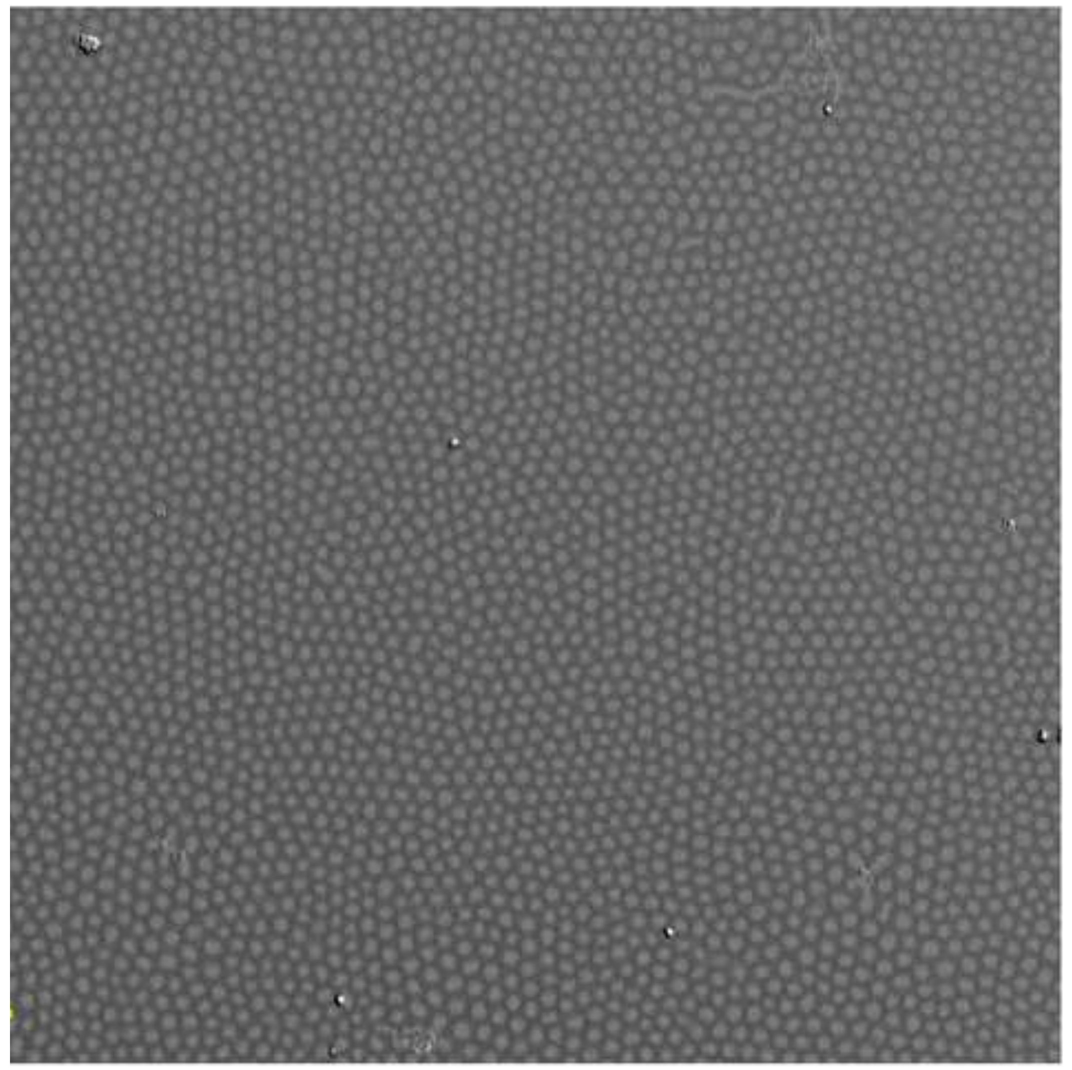
6-atic



Zazvorka et al. Adv Funct Mater 2020

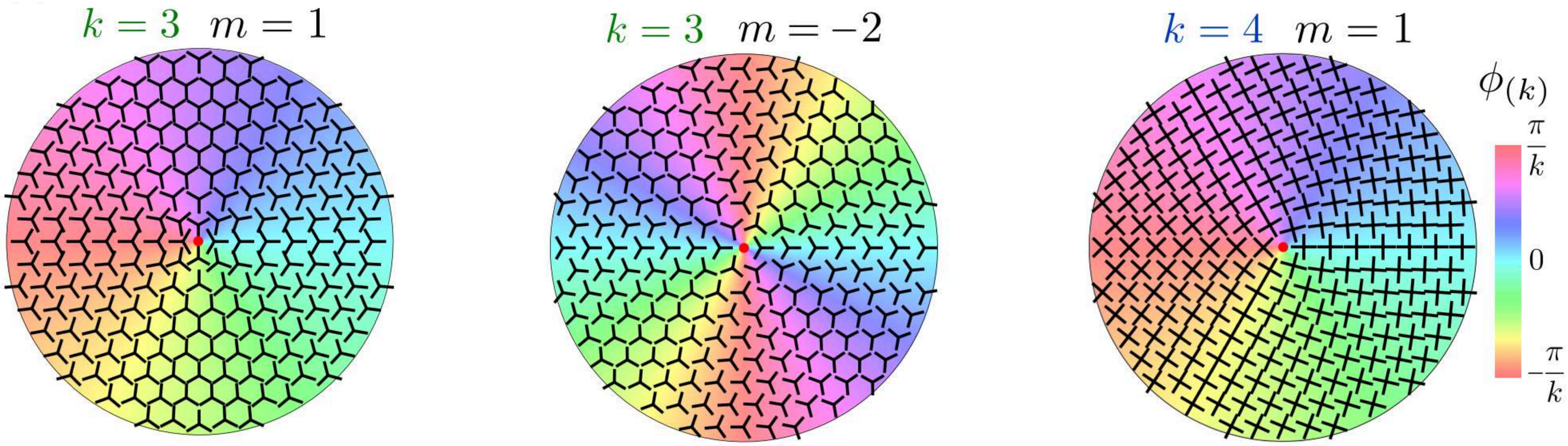
skyrmions

See also: Bowick & Giomi. Advances in Physics, 2009
Giomi, Toner & Sarkar. Phys Rev E 2022

Fractional defects in k-atic LCs

Complex k-atic order parameter
$$\Psi_k = |\Psi_k| e^{ik\phi_{(k)}}$$
k-atic orientation

Topological charge
$$q = \frac{1}{2\pi} \oint_{\mathcal{C}} d\mathbf{l} \cdot \nabla \phi_{(k)} = \frac{m}{k}$$



$q = +1/3$ $q = -2/3$ $q = +1/4$

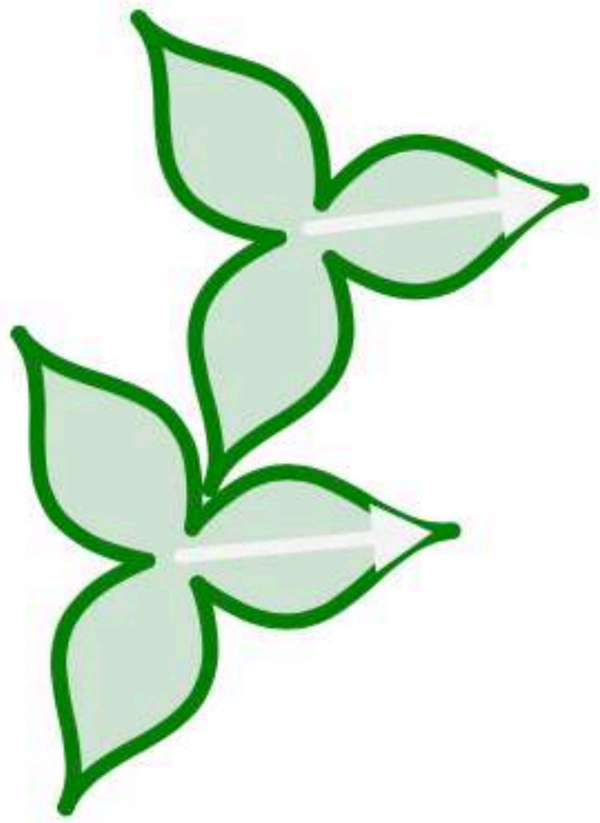
Minimal microscopic disordered-lattice model

Alex Mietke

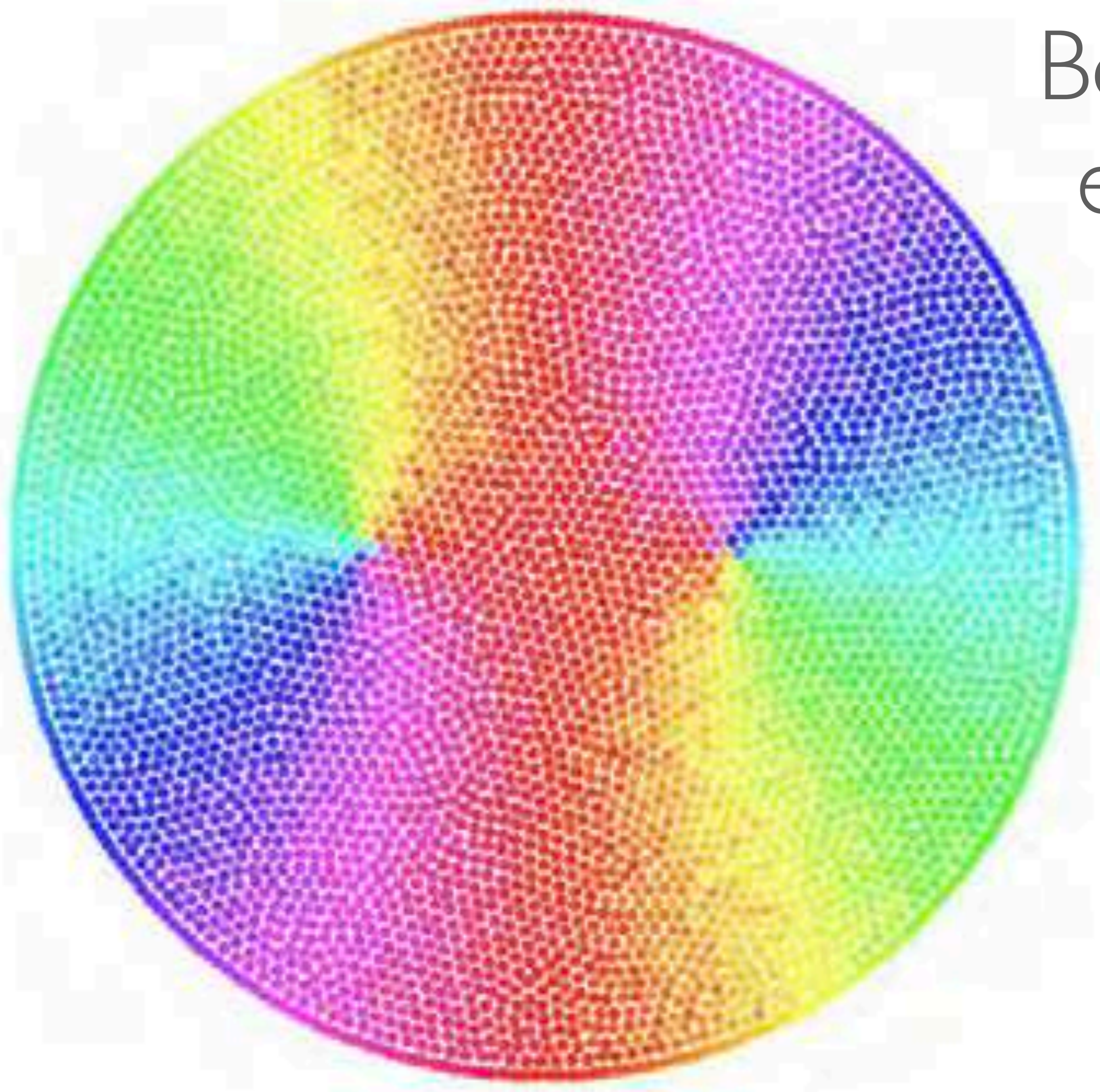
$$\frac{d\alpha_i}{dt} = \frac{g}{\pi R_\alpha^2} \sum_{j \in \mathcal{N}_i} \sin\left[k(\alpha_j - \alpha_i)\right] + \sqrt{2D_r}\,\xi_i(t)$$

g > 0 : alignment

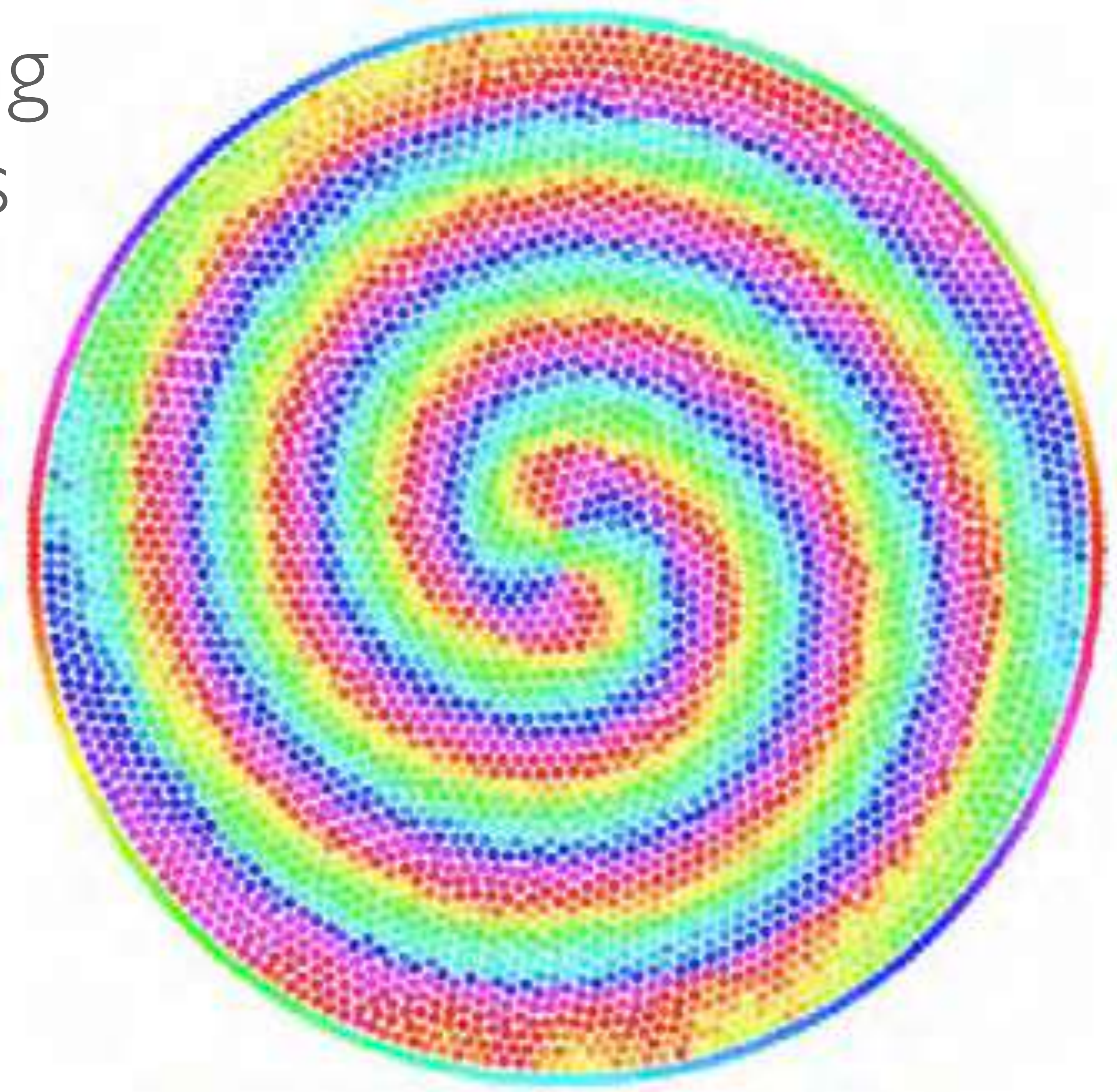
g < 0 : anti-alignment

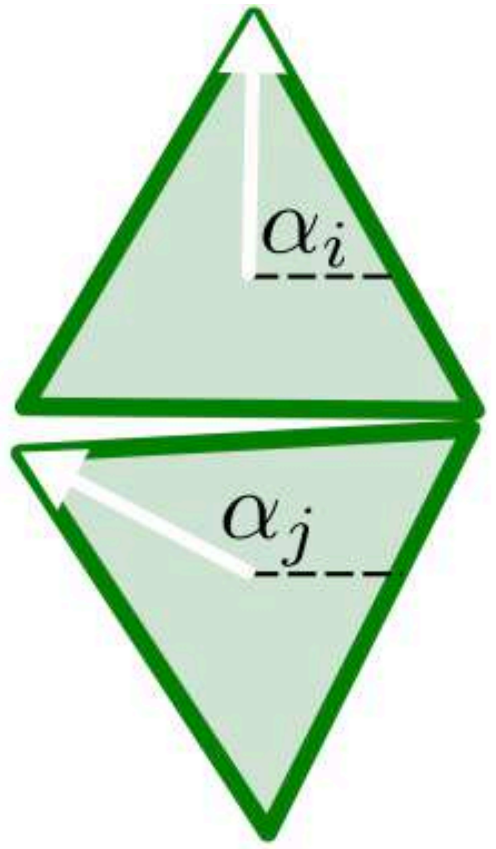


$k = 3$



Boundary anchoring enforces 2 defects

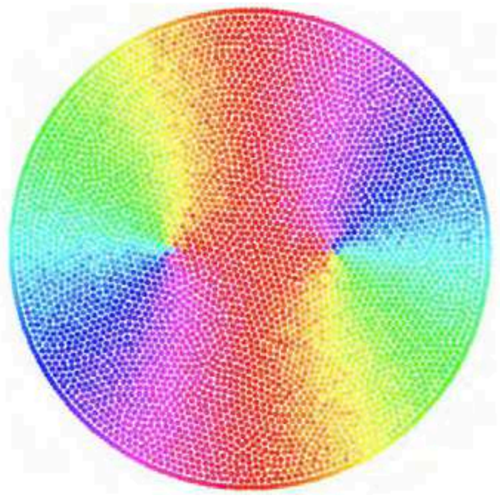




$k = 3$

PRX 2022

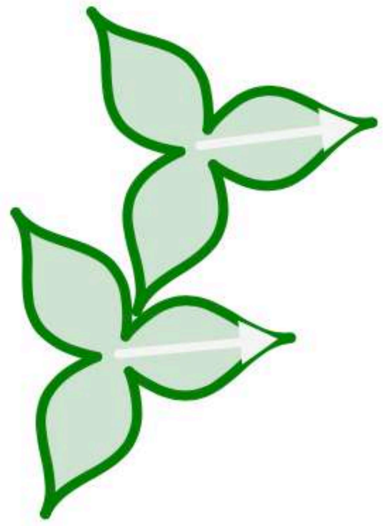
Minimal microscopic random-lattice model

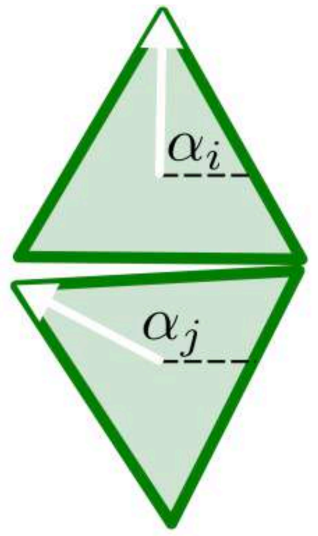


$$\frac{d\alpha_i}{dt} = \frac{g}{\pi R_\alpha^2} \sum_{j\in\mathcal{N}_i} \sin\left[k(\alpha_j - \alpha_i)\right] + \sqrt{2D_r}\,\xi_i(t)$$

g > 0 : alignment

g < 0 : anti-alignment





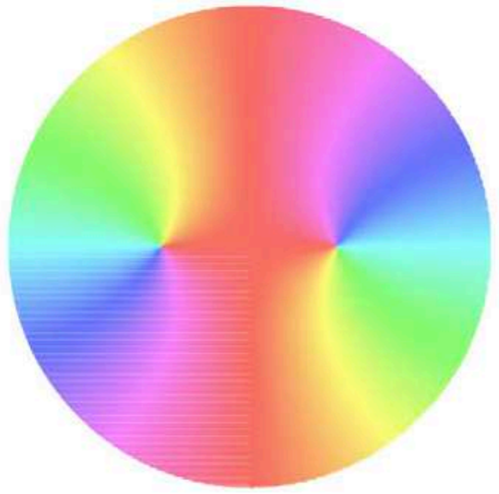
$$\tau\partial_t\Psi_k = -\left(A + B|\Psi_k|^2\right)\Psi_k + \mathcal{L}\left(\nabla^2\right)\Psi_k = -\frac{\delta\mathcal{E}_k}{\delta\Psi^*}$$

g > 0

g < 0

Ginzburg-Landau

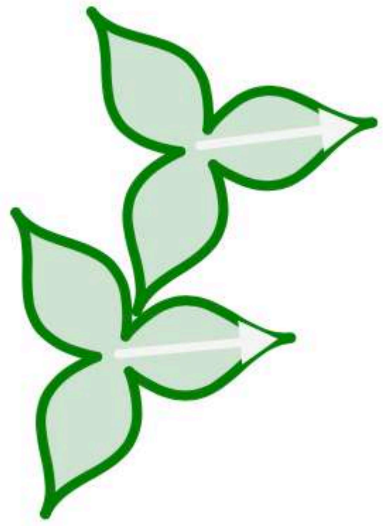
$$\mathcal{E}_k = \int dA\left(A|\Psi_k|^2 + B|\Psi_k|^4 + L^2|\nabla\Psi_k|^2\right)$$

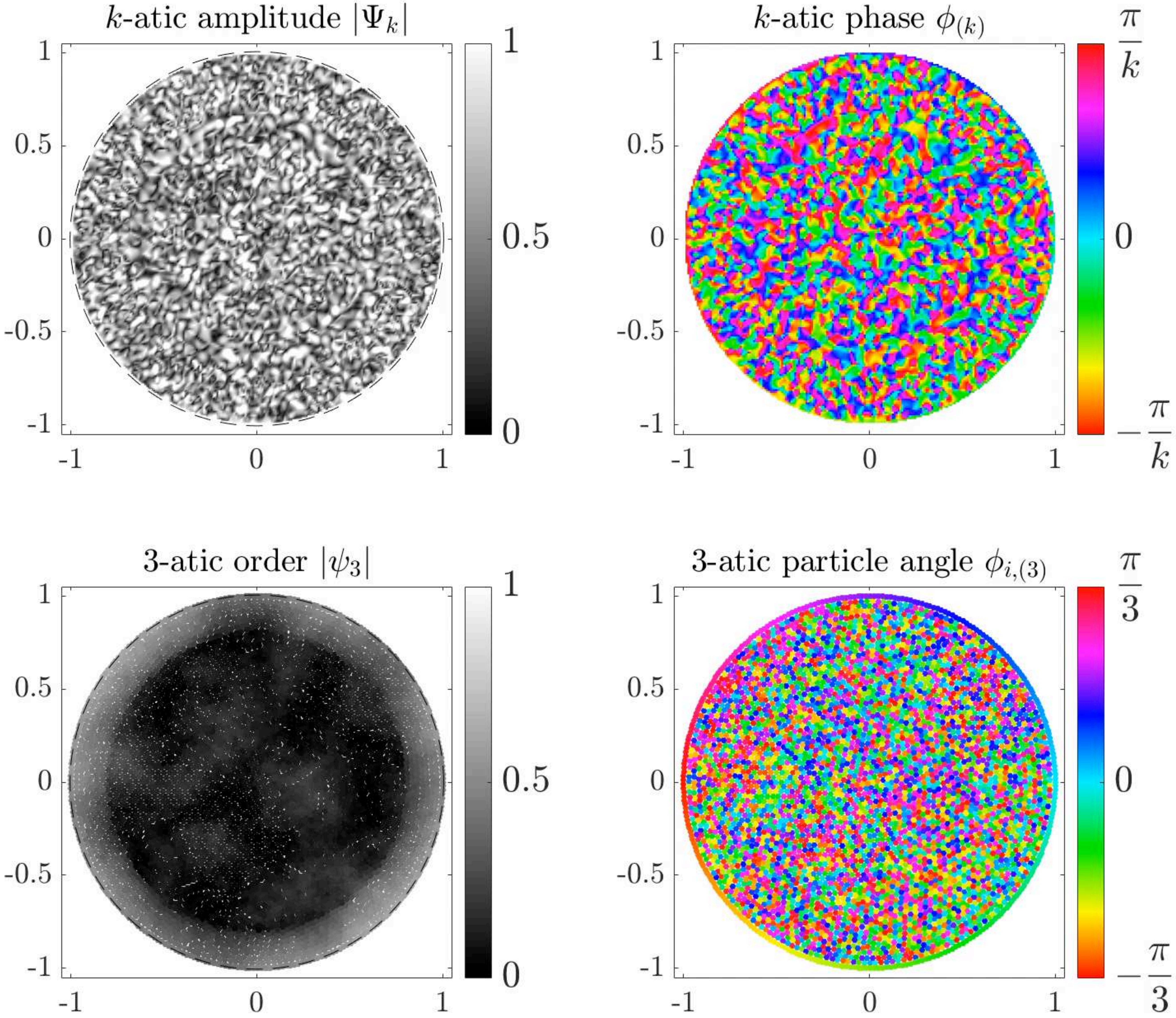


$$\mathcal{E}_k = \int dA\left(A|\Psi_k|^2 + B|\Psi_k|^4 - L_1^2|\nabla\Psi_k|^2 + L_2^4|\nabla^2\Psi_k|^2\right)$$

Landau-Brazovskii-Swift-Hohenberg

Relaxation dynamics

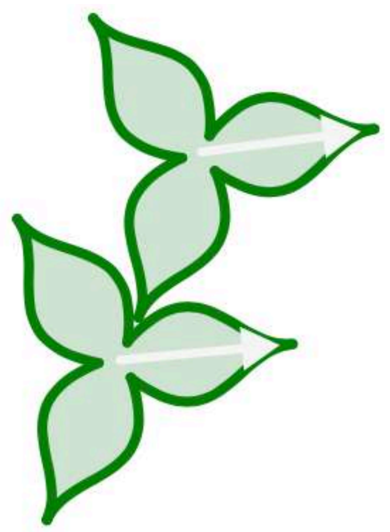


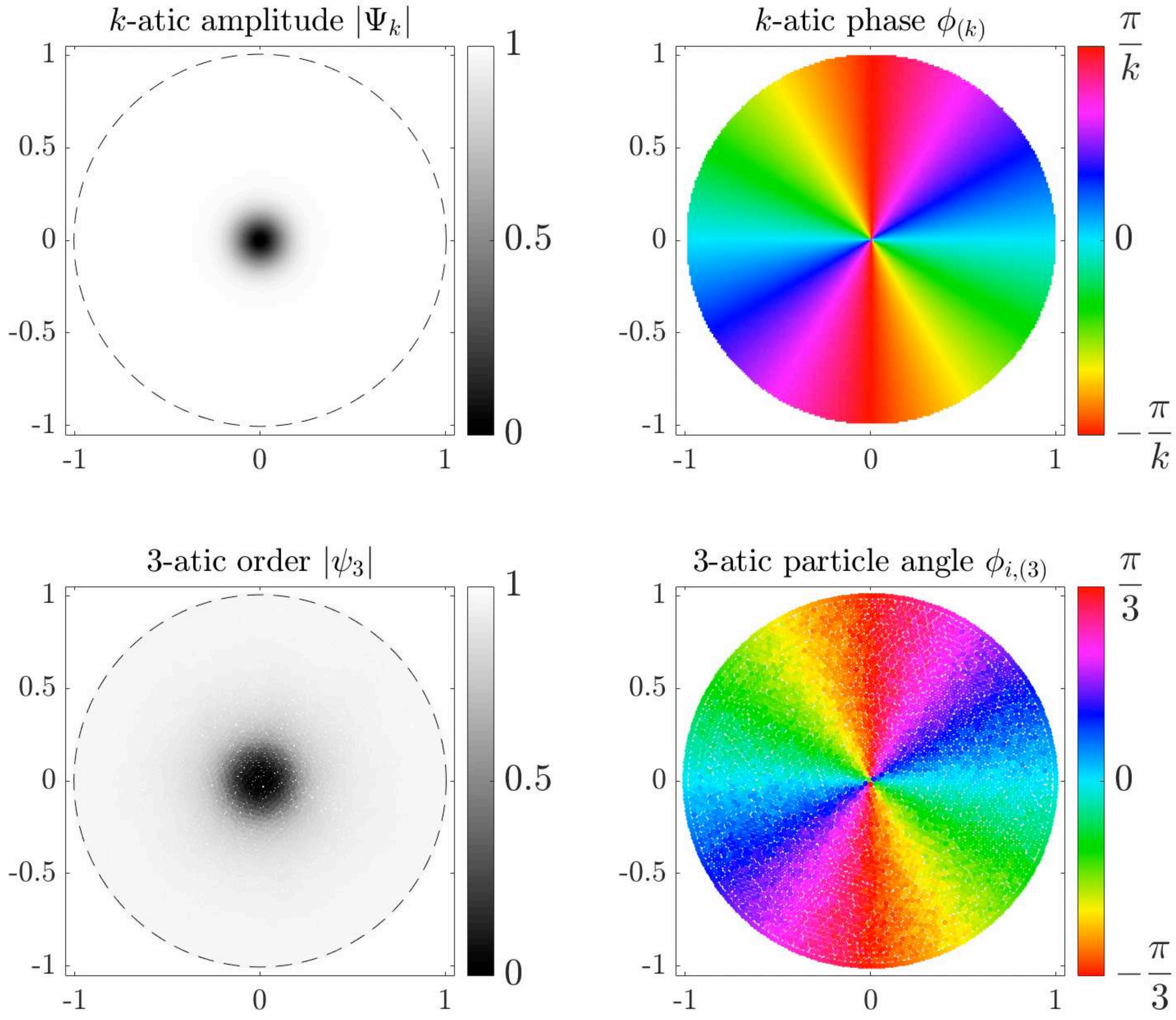


Continuum theory

Particle simulation

Defect splitting



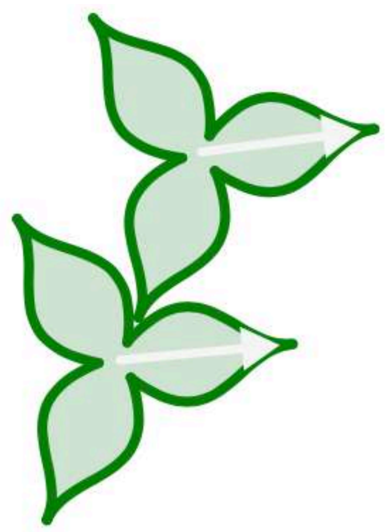


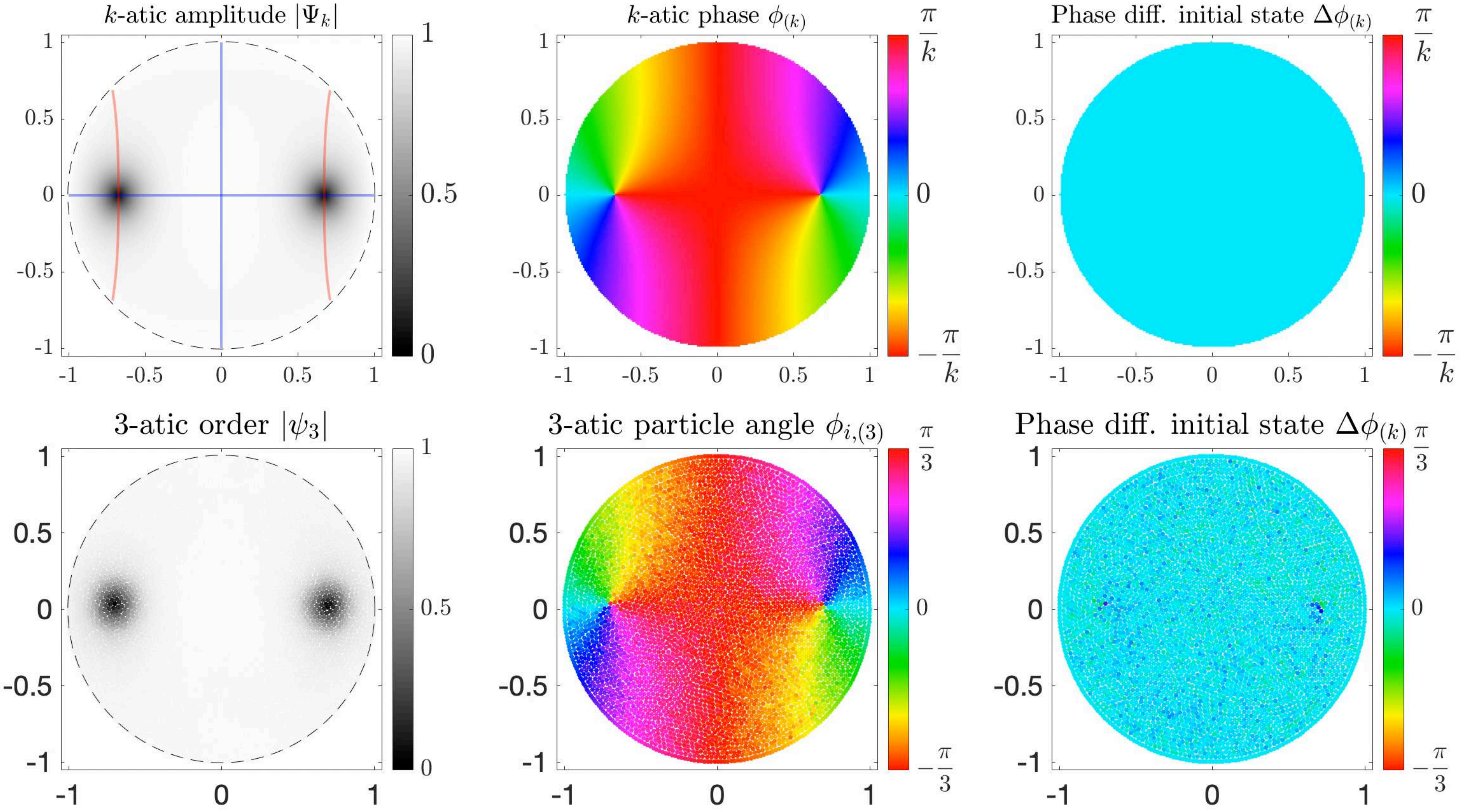
Continuum theory

Particle simulation

Defect braiding

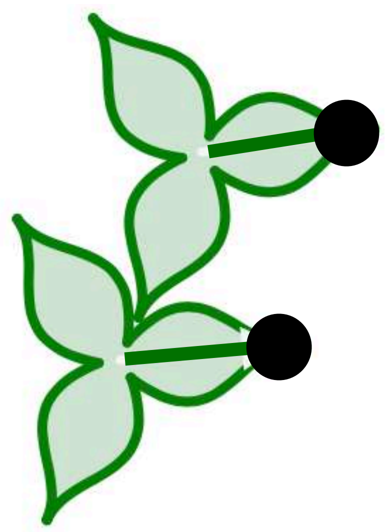
Time-dependent boundary

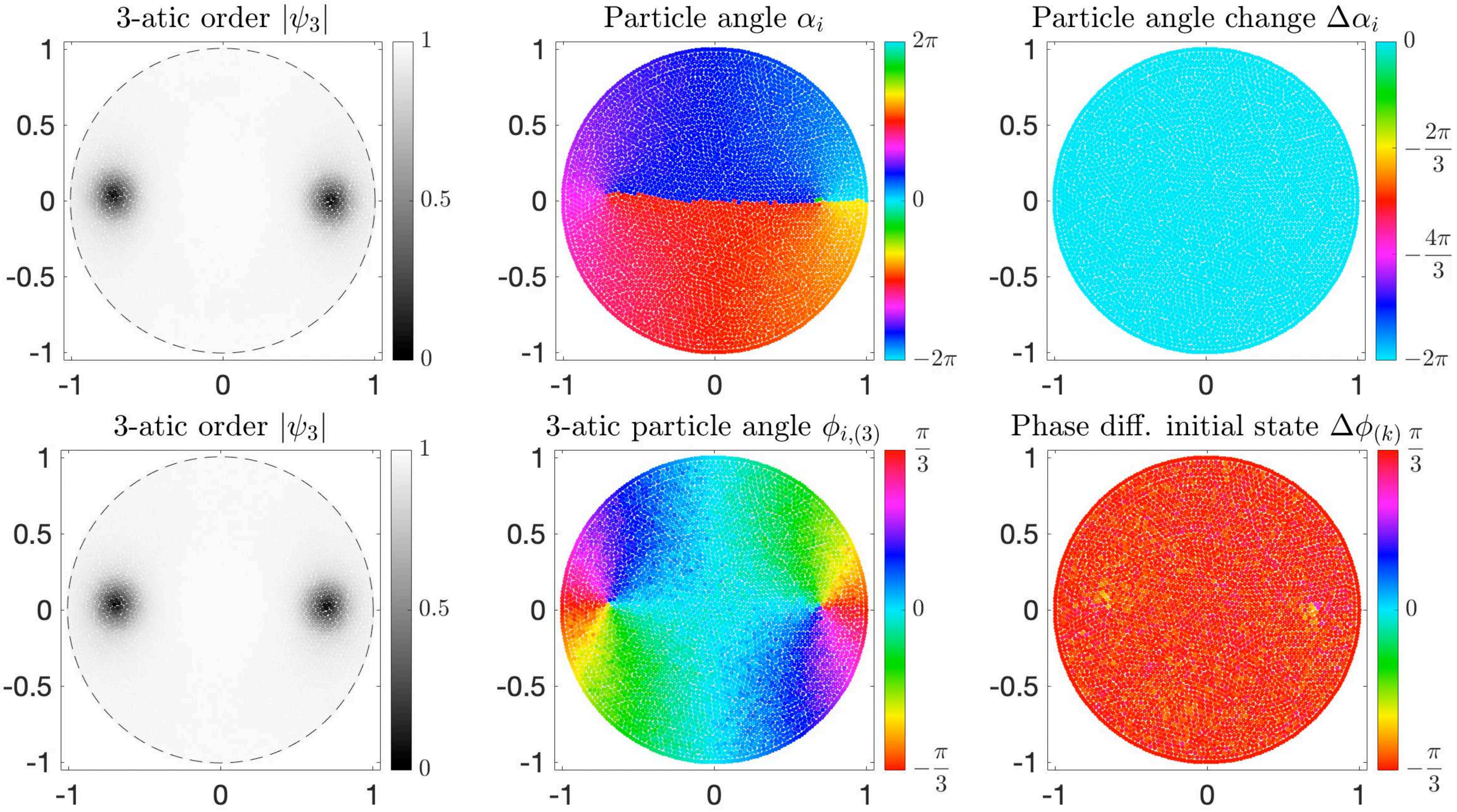




Defect braiding

'anyon-like'



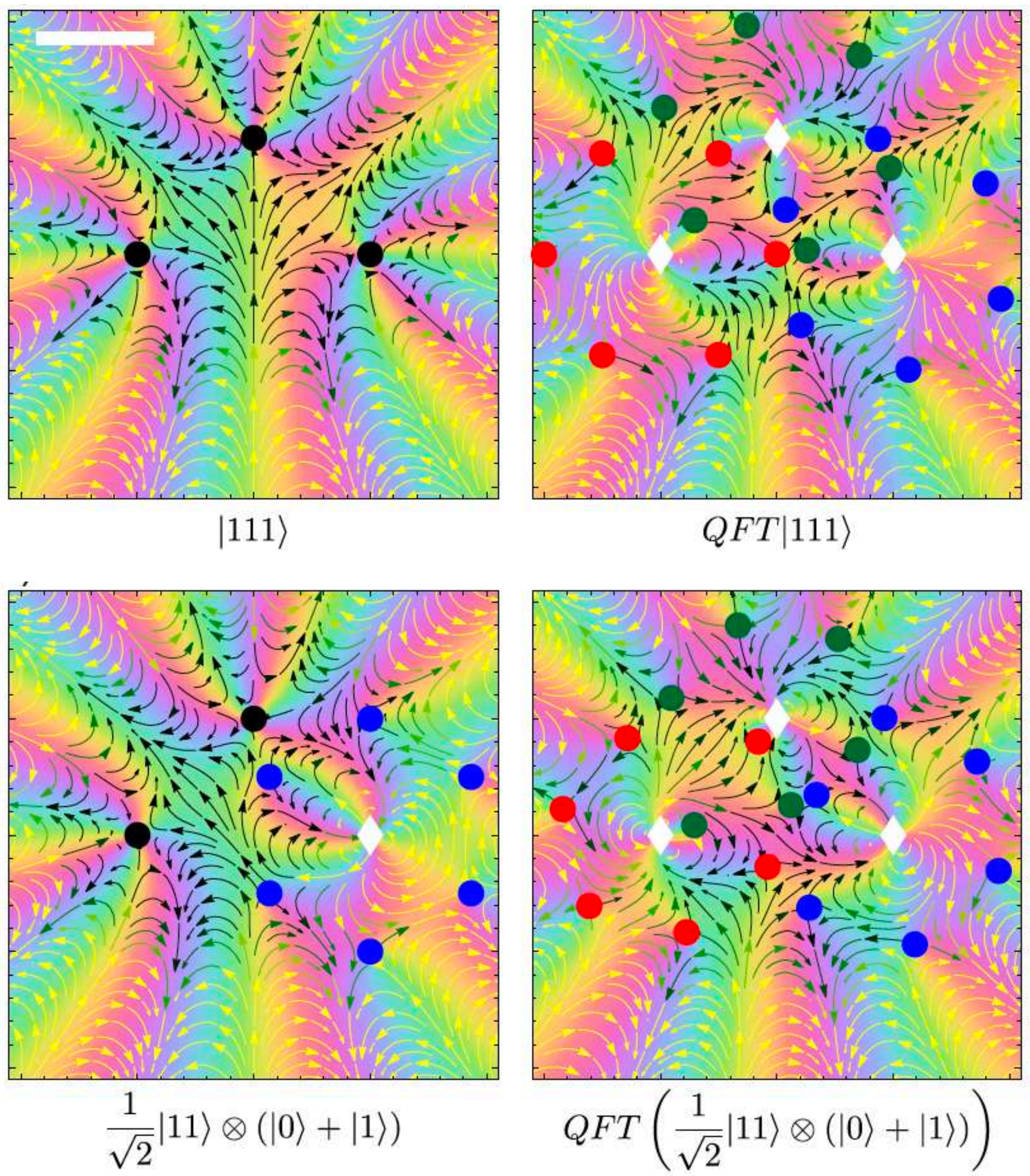


More analogies

PHYSICAL REVIEW RESEARCH **6**, 043039 (2024)

Harmonic flow-field representations of quantum bits and gates

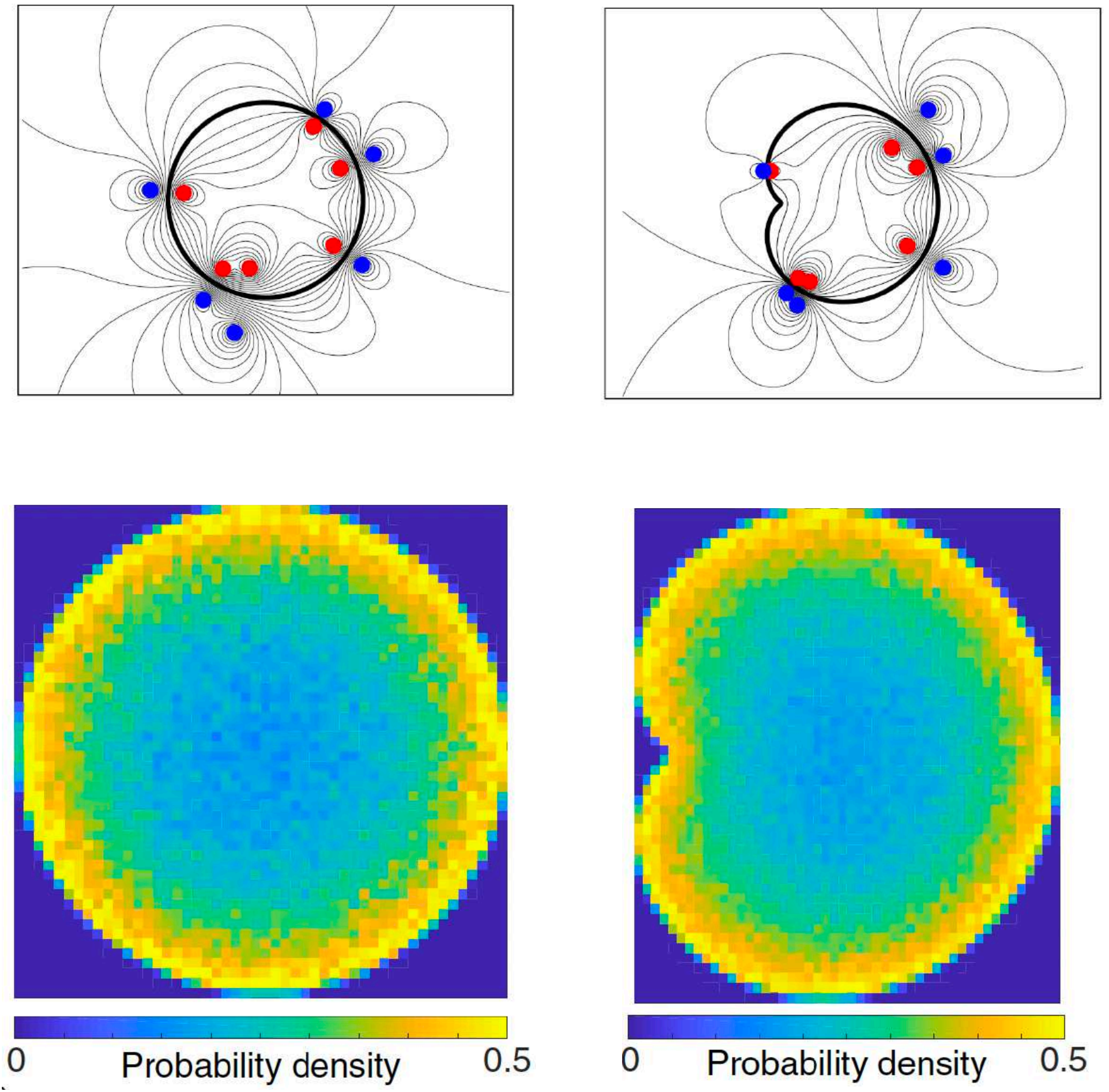
Vishal P. Patil,[1,2,3,*] Žiga Kos,[1,3,4,5] and Jörn Dunkel[1,3,†]



PHYSICAL REVIEW FLUIDS **6**, 064702 (2021)

Chiral edge modes in Helmholtz-Onsager vortex systems

Vishal P. Patil and Jörn Dunkel



Mapping Hilbert space vectors to 2D compressible flow patterns

Real space argument: edge modes from boundary conditions